

THE
MATHEMATICAL
GAZETTE

*The Journal of the
Mathematical Association*

Vol. XLII No. 341

OCTOBER 1958

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THE MATHEMATICAL ASSOCIATION

AN ASSOCIATION OF TEACHERS AND STUDENTS
OF ELEMENTARY MATHEMATICS



*'I hold every man a debtor to his profession, from the
which as men of course do seek to receive countenance
and profit, so ought they of duty to endeavour themselves
by way of amends to be a help and an ornament there-
unto.'*
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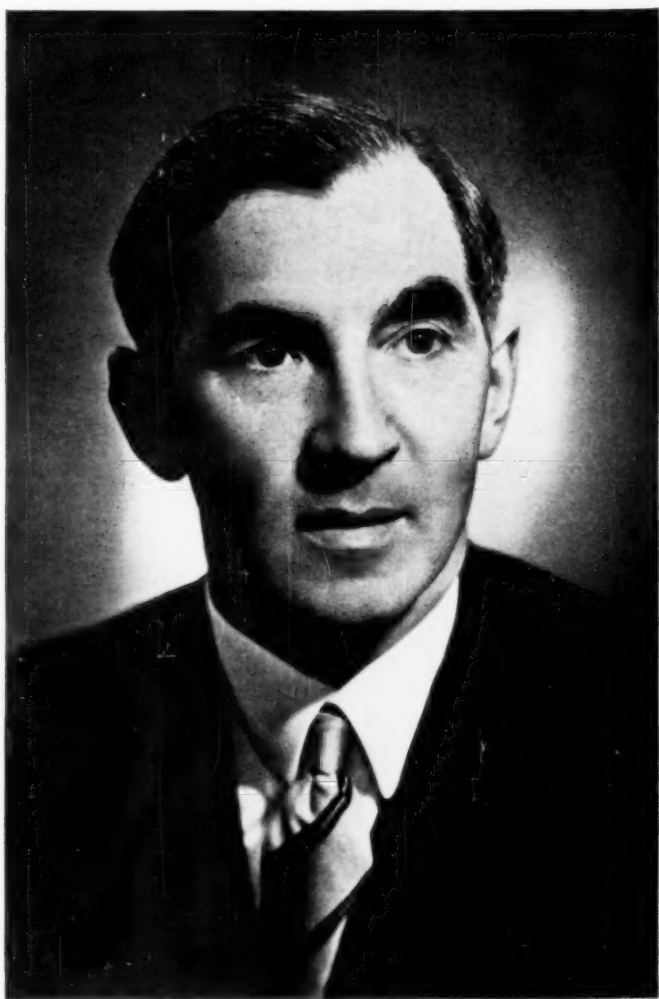
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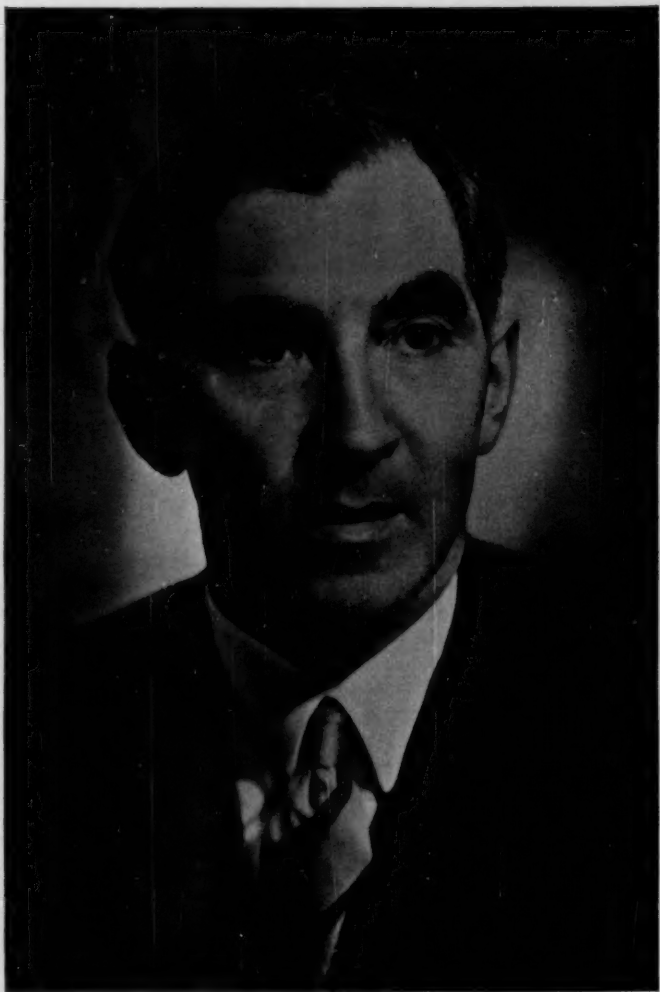
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SECONDARY SCHOOL MATHEMATICS AN INTERNATIONAL SURVEY

Presidential Address to the Mathematical Association

April 1958

By W. J. LANGFORD

INTRODUCTION

Most of us here present today have been involved (facing one way or the other in the class-room) in one of the most far-reaching and important revolutions in teaching theory and practice that any national system of education has ever known. The first step was Rawdon Levett's famous letter to Nature in 1870, though it was thirty years before any solid progress could be claimed. Another twenty years of steady endeavour, in which some well-known members of this Association took a leading part, was necessary before the first of our major reports (that on Geometry) was published. Yet another twenty years, in which time four more reports were issued, bring us to the greatest change of all: the Jeffery Report and the acceptance by the schools and the Examining Authorities of the Alternative Syllabus. So far as can be judged after ten years' experience this revolution, far-reaching in its effect on the mathematical work in schools, has been entirely bloodless—for Euclid, if deposed, is neither exiled nor ostracised.

It is well that we should, from time to time, look back over past achievements, and past mistakes. The realisation of what has been done is an encouragement to further effort and oftentimes a source of inspiration in new fields of progress. There is perhaps a tendency, in which our island situation is a contributory factor, to imagine that we are alone in the progress and the changes that have been made. It is my privilege this morning to bring to you some account of the present standard of mathematical teaching

in other countries. The result, I trust, will be to leave us proud and satisfied with what we have accomplished while still ready to profit from the experiment and experience of our colleagues abroad.

I could not have attempted this task without two experiences both of which came to me, I believe, as a result of my membership of the Mathematical Association. In 1950 I represented the Association at a residential Conference in Holland at which I joined delegates from Denmark, France, Belgium and Switzerland to discuss with the Dutch teachers of mathematics the educational systems, and the methods of teaching mathematics, in our respective countries. In 1956 I was a member of the United Kingdom delegation at the Geneva Conference of the International Bureau of Education, where one of the three items on the agenda was this same topic of mathematics teaching in secondary schools. At both these meetings I had many opportunities for personal discussion, and on this and a multitude of reports I have been able to base what I shall say to you to-day.

THE AIMS OF MATHEMATICS TEACHING

At Geneva 74 nations were represented, and advance information was obtained through 62 national replies to a number of questions about method and content in the mathematical syllabuses of secondary education. One of the most interesting of these was concerned with the reasons for including mathematics in the general curriculum, and the purpose underlying its teaching. Almost every country, as one would expect, indicated that it was intended to "lay a sound foundation of mathematical knowledge for pupils intending to follow scientific or technical studies," and pointed out that there is an ever-increasing number of professions which demand a deeper and more active acquaintance with the subject. Some two-thirds of the replies indicated the need for a facility in the elements of mathematics as a means of solving "the everyday problems of living." Some laid it as a duty upon the individual citizen to acquire a sufficient technique as would enable him to "participate in his country's economic affairs." The more interesting ideas, however, came from the relationship between mathematics as a subject in the curriculum and the aims of a general education. In every country the subject is regarded as one of major importance (in some cases it is given a weighting of 2 or 3, in comparison with other subjects, in leaving and promotion examinations). It is said to "engender an appreciation of the value of the exact sciences and the part which they have played in man's evolution," to "impart the hall-mark of a sound education," and to act as a "source of cultural enrichment."

The theory of the "transfer of training," now largely discounted

here, still holds its own in many countries, and this attitude is generalised as "the cultivation and ripening of moral and intellectual faculties contributing to the development of personality." Hungary hopes to achieve "capacity to understand and formulate the laws governing the environment," Spain to develop "intelligence through logical thought," Thailand the "capacity for concentration, accuracy, rapidity, orderly thinking and confidence," Roumania looks towards an "awareness of order and beauty," the Philippines expect an "awareness of the value of mathematics as a cultural heritage and as a factor in human progress," Italy moves into the moral field with "orderliness, exactness and soberness in the use of words" while the French believe that the "aims of mathematical teaching are the aims of secondary education as a whole, including the formation of an élite class."

THE MEANING OF "SECONDARY"

The countries represented at the Geneva Conference formed such a worldwide selection that one would expect to find great variations in the conception of the secondary course. It has been interesting to analyse it in two ways:

- (a) by the age of the pupils at the *end* of the full course available (and here there must clearly be some overlap into our idea of further education) and
- (b) by the length of the course.

(a) Cambodia gives 21 as the end of its secondary cycle, while in Laos and Syria the age is 20. In countries which have a closer parallel with our own system we find courses ending at age 19 in 8 countries, at 18 in 31 countries, and at 17 in 15 countries. Burma with a 5-year course, Nepal and Spain with 6-year courses, all end this cycle at 16, and Pakistan has a 5-year course between the ages of 10 and 15. I have been unable to obtain the age in the case of the Philippines but the course there is one of 4 years.

(b) Two countries, Denmark (age range 15 to 18) and Egypt (14 to 17) limit their secondary course to 3 years; 8 countries have 4-year courses, 14 have 5-year courses, 21 have 6-year courses, and 12 have 7-year courses. Finland and U.K. (11 to 19) and Italy (10 to 18) have 8-year courses and the Federal German Republic and Sweden have 9-year courses between the ages of 10 and 19.

No one will now expect to find anything like agreement in the contents of the various mathematical syllabuses even within the upper cycle of secondary education. All that I can hope to do is to indicate some comparisons, and some contrasts, with the work which we normally find in the advanced mathematics courses in this country.

PLACE OF MATHEMATICS IN THE SECONDARY SCHOOL CURRICULUM

Beyond all doubt mathematics occupies a definite, and important, place in the secondary school curriculum in all the countries submitting reports. In some cases there is a distinction between "arithmetic" and "mathematics" in the earlier years, but only two countries (Philippines and Vietnam) restrict even the first year of the lower secondary cycle to nothing but arithmetic. For the rest mathematics is obligatory throughout the earlier years in the secondary school and something between 10% and 20% of the weekly time-table is allotted to its teaching. In some two-thirds of the countries mathematics appears as an obligatory subject throughout the whole course in the secondary school, and we find that in about half the remaining countries mathematics is an optional subject in the upper cycle; in the rest it is obligatory in the science sections of the course. Only rarely is a deliberate distinction made between girls and boys: in Denmark's *realklasse* mathematics is compulsory for the boys and optional for the girls, while in India the girls are required to study only arithmetic.

In countries where the course is obligatory and the syllabus is imposed on the schools by the appropriate Ministry, it is usual to find that arrangements are made for those of greater ability to extend their knowledge by membership of mathematical societies which meet out of school hours. In the U.S.S.R. national competitions known as "olympiads" are a significant part of this extra-curricular work and a similar practice has been in operation in Hungary since 1894. Where mathematics is an obligatory subject only in the science sections of the later years, we naturally find an increased allocation of time, and the range now varies from 20% to 40%, or even more, of the whole of the time-table.

THE EARLY YEARS

The Education Act of 1944 has widened a gap by the enforced removal of primary age-groups from some grammar and other secondary schools. The coordination of the work done in the later years of the primary school with that done in the first year courses in secondary schools is of vital importance to the teaching of mathematics, and in this country at present we are doing little or nothing about it. It has been alleged that the failure on the part of some children to make progress in the secondary syllabus is a direct result of unsatisfactory methods in use in primary schools. The Association's recent report on Primary Schools will no doubt have done much to remedy this defect, but a big responsibility rests upon those teaching in secondary schools to see that the work in their first-year courses constitutes a challenge to the pupils.

These young people have undergone a profound transformation

at a most impressionable age: in general they have moved from small schools (in which they have been the seniors) into much larger schools where they are very small fry: they change from a system in which almost all the work is in the hands of one teacher to a specialist system for almost every subject in the curriculum: they have looked forward keenly (some perhaps with anxiety and dread) to this change which is the culmination of the over-publicised Junior Leaving procedure: in some cases parents have been a source of emotional disturbance through their anxiety about so-called "success" or "failure," while in the minds of some people the success of a primary school is still regarded as measured by the proportion of its children which achieves the grammar school grade. All this persuades the child that he is really on the threshold of an entirely new kind of educational life, and in some respects this expectation is realised; he begins a foreign language, he uses a gymnasium and a crafts shop, he really sets foot in a laboratory—but in arithmetic (which is now called "maths") he often goes back over much of the work he did in the primary school. This so-called revision of the earlier work is entirely misconceived involving as it sometimes does unlearning one method of (say) subtraction to meet the whim of the master who insists on "borrowing from the top line" instead of "paying back to the bottom line." We simply must see to it that the arithmetic course breaks new ground at once: there is a good case for restricting the scope of processes now taught in primary schools but the coordination of the work in the two kinds of school is our responsibility, and is another aspect of our freedom that we should do well to handle with the greatest care. Our colleagues in the primary schools need and deserve a lead from those working in secondary schools and it should require little more than common sense and understanding to achieve it.

In Bulgaria "with a view to facilitating the transition ... the secondary teachers have made recommendations to primary teachers concerning methods ..." It is realised in Columbia that the "approach is fundamentally different from that ... in primary schools where the teacher's dominating position tends to make teaching methods authoritarian." France claims that "care has been taken to ensure the continuity of mathematics syllabuses and teaching methods in the period of transition from primary to secondary education," and similar claims are made for Hungary and Laos. Shall we in this country, where a great revolution in secondary teaching methods has already been achieved, neglect this vital bridge in the path of progress? We shall do so at our peril, but I am certain that if we can but realise the vital importance of this consideration the necessary action can be taken within a school generation.

SYLLABUSES

In many countries there is clear evidence that the whole content and presentation of school mathematics has recently been subjected to careful scrutiny: a number of others, possibly influenced by what is going on around them, state that revision is intended in the near future. The reasons given for these changes are often to be found in the aims expected to be achieved by the subject within the school curriculum, and there is a clear desire to bring secondary mathematics teaching in line with the recent educational developments. Emphasis is placed upon the need for the pupil to understand the concept rather than on the mere acquisition of a mechanical skill, and it is hoped to achieve this through examples of a practical nature. There is here, of course, a desire to use the most modern methods of teaching and also to derive the maximum assistance from models, films, film-strips and the other sensory aids which have become part of the teachers' normal stock-in-trade. There is also an indication that the very development of mathematics itself has had a profound effect on the scope of the work in the schools. Here is an aspect of the problem to which our own Teaching Committee has given a great deal of attention. Whether or not the boundaries of mathematical knowledge are being pushed back at a greater rate than was the case in the last few centuries I am not qualified to judge, though the fact that the content of the subject as a whole is far greater than it was a century ago makes it reasonable to expect that there are also more opportunities for original work in a variety of fields. It is certainly clear that if the lecture courses in our universities are to carry graduates within reach of some of the present limits of knowledge, the equipment of the undergraduate fresh from school must either be vastly greater than it was in the last century or else be differently conceived: to this aspect of the problem I shall return later in this paper. About one third of the countries commented upon the way in which it was thought that this development had influenced their own syllabuses. The most notable modifications include the earlier introduction of calculus, coordinate geometry, statistics and vectors, as well as the revolutionary changes which spring from a consideration of functionality.

I can, unfortunately, do no more here than select a few of the unexpected items from national syllabuses as examples of the way in which this development is proceeding. The Afghanistan syllabus includes an item "Thales' theorem and its applications." The context seems to indicate that this means the equality of the ratio of corresponding sides in equiangular triangles, though some quick research, recently undertaken in the tea-interval of the Analysis sub-Committee, revealed that Thales is thought to have

been the first to "prove" a geometrical theorem in the sense we now understand, and that he is thought to have been responsible also for the discovery of the base-angle equality of the isosceles triangle as well as the value of the angle in a semi-circle. In this syllabus we also find two unusual entries: "inequations of the first degree" and "finding two numbers given their sum and their product." The Belgian syllabus includes the theorems of Menelaus and Ceva in the 4th year (age 15/16) and those of Pascal and Brianchon in the 5th year. We also find in this course "the theory of errors" and "the number of factors of a number." In Byelorussia "an introduction to the principle of axioms" forms a specific part of the school course in mathematics and "history ... and the lives of famous mathematicians of the U.S.S.R. and countries abroad" is included. The seven-year course in Ceylon contains little more than our normal arithmetic, algebra (to the solution of quadratic equations) and geometry (to Pythagoras' theorem). The Dominican Republic includes the "higher forms of calculation" and "Newton's formula" in its syllabus, while Ecuador has a curious item called "double radicals." From the Finnish syllabus we notice that the derivative is applied to "second degree trinomials" and that some attention is given to "irrationals and imaginary numbers."

In France a circular issued in October 1952 stresses "the necessity of giving some attention ... to the ways in which the various discoveries were made ... which offers the only means of showing how creative thought, experimentation and systematic research have led up to the knowledge of to-day." Within the German Federal Republic the various Länder are autonomous in the prescription of the content of syllabuses but in most of them "increasing emphasis is being given to the notions of displacement (translation, rotation and revolution of the plane of projection) and to the method of affinities." In the last year (age 17/18) of the course in Hungary the pupils are given an "introduction to analysis" and to the "whole and fractional roots of equations of the highest degree." In Israel many schools devote two periods a week in the 3rd and 4th years (ages 16/18) to the "history of science including mathematics." Japan has a very carefully graded mathematics course in the three years of the upper cycle. Those who obtain 6 credits in Mathematics I during the first two years (ages 16/17) pass on to course II and those who need it will do course III in the last year. This syllabus includes "sequences and series, calculus, probability and statistics" as well as "complex members and their geometrical representation." In the Luxembourg syllabus for boys' schools we notice that those in the Greek-Latin section include in their mathematics course "the limit and continuity of a function," "the concept of the infinite integral,"

"homologous transformations" and "the intersections of two surfaces." Nicaragua requires its pupils to deal with "irrational and biquadratic equations" in the second year (age 14/15) and the "power of a point in relation to a circle" in the third year. In addition to a specific mention of history and axioms, the Philippines course has the attractive item "practical problems concerning thrift in the home, the community and the nation etc."

The Polish scheme of work is very emphatic about the teaching of the history of mathematics and the logical development of the subject. Here we find "the mathematics syllabuses ... include an introduction to the history of mathematics showing the dependence of the evolution of mathematical concepts and symbols on the development of man's creative powers." In the 4th year course in Portugal (ages 13/14) an unusually early discussion is included on the "concept of the infinitely small and infinitely large" while two years later consideration is given to the logical complications of "symbols signifying impossibility and indeterminateness." From the Roumanian syllabus comes the sixth year entries (age 16/17) of "systems of indeterminate equations of the second degree" and "the solution of binomial equations of the third, fourth and sixth degree." This syllabus is the only one to mention non-Euclidean geometry specifically, saying that teachers "take special care to make their pupils understand the principle of axioms including Euclid's axiom of parallel lines, and also that of Lobatchevsky."

METHODS OF TEACHING

The freedom which we enjoy in the schools of this country to frame our own syllabuses, to choose (and even write) our own textbooks is greatly envied by teachers in most other countries. When we remember the campaign of Rawdon Levett, Siddons and Tuckey and our other distinguished predecessors in the Association for the Improvement of Geometrical Teaching against the "tyranny" of Euclid's order and the philosophical (as opposed to the mathematical) approach to geometry which this engendered, we should at one and the same time be grateful for the fruits of their labours and careful lest our very lack of restriction should make us careless. We in the schools have a duty to see that the mathematical courses we devise for our pupils are appropriate to their needs and understanding and that adult complexities are not a source of confusion and defeat to the young mind. We must also ensure that the beginnings of a structure gradually emerge. It is upon these foundations that our colleagues in the universities must begin to build and, if we have attempted to construct the roof before the walls are up (or, to push the analogy a little further, if our walls are out of "true") then the whole construction must collapse, though,

unlike Samson, we shall not share in the fate of those upon whom it falls. I am convinced that a good deal of the undergraduate failure in mathematics about which we hear is due quite as much to misplaced enthusiasm in the schools as it is to the real incapacity of our students for advanced work. Whatever we teach must be soundly based so that sufficient scaffolding supports the intuitive experience with which of necessity we must commence; far better to build only part-way than to reach the sky and to perish in the rarified atmosphere of logical inconsistency.

In over half the countries mentioned the syllabuses are prescribed by the state departments and official instructions about method are issued for the guidance of teachers in the schools. Moreover, in many cases permission will be given to teach a subject only when a course in methodology has been successfully followed after the intending teacher has himself reached the necessary level of competence in his subject. It was for me a source of great pride and satisfaction to read in the report of the United Kingdom "There are no official instructions concerning methods of teaching mathematics, but pamphlets of suggestions have been published by the Board of Education. The reports published for the Mathematical Association are the chief guides in this connection." On my return from Baarn, I was glad to use the kind help of the Netherlands Embassy in sending a large consignment of our reports to the Dutch teachers whom I met at that Conference.

An even closer control is exercised over textbooks than over syllabuses or methods. This may be achieved in one of several ways: the Ministry may publish a list and the teacher's selection is limited to the texts appearing in it; the teacher may be required to submit his list of recommendations for the approval of the education authority: in other cases the textbooks are standard throughout the country, they are published centrally and then are issued to the schools. In only about a quarter of the countries is the choice entirely at the discretion of the teacher and the school.

In Belgium endeavours are being made through a mathematical journal to replace the traditional treatment by more modern methods but teachers are "not allowed to abandon the textbook and dictate long notes to their pupils." The instructions issued in Byelorussia "help to promote a scientific attitude in the pupils and to lead them towards a global conception of phenomena. Mathematics is presented as a first-class instrument for the investigation of the material world, leading both to the study of natural phenomena and the discovery of means of controlling them." With each textbook issued for use in Czechoslovakia there is a "companion hand-book for teachers explaining the system and distribution of the subject matter in the textbook and making recommendations as to methods."

In Ecuador the "basis of mathematical reasoning is the concept of 'function.'" It is to this concept, therefore, that pupils must be introduced at first in a concrete and objective way." Some of the books used in Egypt where the Ministry of Education controls the texts are the "result of competitions," and "others are chosen by commissions of experienced teachers and inspectors." From the French recommendations I select the following delightful extract "to begin the study of a given question only when the whole class can follow it: clearly and honestly to indicate points which at a given level must be admitted without demonstration; to avoid giving anticipatory 'explanations' under the pretext of 'opening a window' which is obviously above the pupils' heads: to forgo all demonstrations, even if deemed 'beautiful' which are far-fetched or too involved, graceful presentation being desirable only if it does not militate against clarity." In Greece textbooks are selected competitively; "authors are required to submit their books to the higher education council which chooses three of them for award, and has one of these three published by the official schools publication service and made compulsory." Our colleagues in Guatemala take no chances about method for we read "... the methods employed include the method of proceeding deductively from the theoretical concept to the practical application, that of proceeding inductively from the concrete to the theoretical, and that of combined induction and deduction."

Israel looks forward confidently to the possibility that the use of films may bring about sensational changes in the teaching of geometry. "The notions of the elements of space are still taught abstractly and theoretically as was the case generations ago. The education authorities are convinced that the introduction of the film ... could greatly affect, if not revolutionise, the results of such teaching." In Luxembourg the medium of instruction in the primary schools is German, and when the children move on to secondary education this language is replaced by French so that the insistence on the use of official textbooks, syllabuses and instructions on method is perhaps easier to understand. It is claimed that "as a result the lessons given in one school are in complete conformity with those given in any other school." Great stress is laid upon the need to achieve understanding and it is recorded that in mathematics "more than any other (subject) learning means understanding. Teachers should ensure that all abstract ideas have been properly understood." An unusual approach is found in Nicaragua where "the teachers generally adopt a theoretical and rational approach, make almost exclusive use of analysis, and do not give any special attention to practical conclusions." In direct contrast mathematics is taught in Peru

"with a definitely practical bias, and the pupils are given practical exercises to do at frequent intervals." No doubt with a view of furthering the practical application of mathematics the use of the slide rule is compulsory in the upper classes of Swedish schools, and official instructions indicate how teachers may "prepare for the theoretical treatment of certain aspects of the subject through giving practical exercises."

I have studied in some detail the position of mathematics and the methods of teaching, in a number of countries. This morning, time forbids me to mention more than one, and I have chosen the Netherlands because of the unique opportunity I was given in 1950.

Mathematics is a compulsory subject through the whole course in all types of Dutch secondary school but one; the A (or languages) course in the modern secondary schools (known as H.B.S.). The syllabus, which is government controlled for all schools, presents little which is unexpected: trigonometry and geometry find a full place; calculus was introduced in 1937, but even in 1950 it had not appeared in the final examination papers. The approach to mathematics is essentially the Euclidean process of logical development from definitions and axioms; only descriptive geometry (by which we mean geometrical drawing) appears in the H.B.S. course, but there is some analytical geometry in the *gymnasium* syllabus.

Education is entirely free in all kinds of school in Holland and textbooks, which may be chosen without restriction by the teachers, are also provided free of charge. There are two associations of mathematical teachers, the Wimecos (which represents those in the H.B.S.) and the Liwenagel (for those in the *gymnasium*). These cooperate with a firm of publishers in the production of a periodical Euclides which is now in its 33rd year. The Institute of Education at Utrecht under the direction of Dr. Bunt is engaged in a good deal of statistical research work on all aspects of the teaching of mathematics.

The Science sections of the H.B.S. are responsible for nearly 60% of the university population in Holland, and the same sections of the *gymnasium* for another 20%, but even with a professional training scheme which is not obligatory, there is a drastic shortage of teachers of mathematics. In 1950, 22% of the teachers had less than statutory qualification, and were being employed under a special licence. An emergency training scheme of 15 months duration was started in 1955, and this produced some 2,000 teachers for the early part of the following year; even so the proportion of unqualified teachers was still as high as 16% in 1956.

An investigation in mathematics was carried out by Utrecht in 1949 with the middle classes of a number of schools. The teachers were given detailed instructions concerning the work to be taught

and the method of teaching, and the results were assessed by a special examination. As the report puts it: "a high percentage of insufficient marks ... will always be, for the teacher, a reason for ... surveying his work, his students, the curriculum or the testing means," and it comments that at least one of these must be at fault. Quoting as axiomatic the proposition that "in order to obtain a reasonable ... success in ... secondary mathematics, no other standard of intelligence is required than that necessary to obtain a reasonable ... success in the ... remaining school subjects," Dr. Bunt goes on to analyse the actual results and concludes that to deal adequately with the syllabus as now constructed the classical grammar schools should increase the time allotted to mathematics by 40% and the H.B.S. by 24%. Where this immense increase could come from is difficult to foresee, for out of 167 hours of instruction, which represents the total of one typical week out of each of the 5 years of the H.B.S. B course, 57 are given to their four languages, 25 to mathematics and 34 to science, leaving only 51 for all the other subjects. It is perhaps as well that it is their problem rather than ours!

CONFERENCE RECOMMENDATIONS

The delegates at Geneva produced a set of recommendations which was subsequently circulated to the Governments of all the countries taking part in the Conference. There is little in the 36 clauses of this document that is not already a normal part of the school routine in this country, and from these I select a few as illustrations: "Mathematics should be regarded as an essential part of the education of a modern person;" "Instruction in mathematics of a cultural rather than a purely technical kind should be offered as an option ... where mathematics is not a required subject;" "The difficulty and the extent of the subject matter ... should be related to the mental age ... and to the pupils' interests and needs;" "... care should be taken not to discourage the less gifted pupils by imposing on them subject matter too complex for their intellectual ability;" "to convert into skills only those processes which have been fully assimilated;" "to develop the subject heuristically rather than to teach it dogmatically;" "to cease putting its various branches into watertight compartments;" "audio-visual aids ... play an increasingly large part in teaching, and advantage should be taken of their use to enable pupils to acquire mathematical abstractions actively."

We should, I think, agree with all these opinions and would believe that, in general, they are practised in this country. When we come to the sections concerning teachers, however, the emphasis is differently placed: "in mathematics ... more than in other

subjects ... the teacher is of prime importance;" "Teachers ... should have studied mathematics at a level considerably beyond that to which they will be required to teach;" "adequate professional training ... should be regarded as a necessary complement of a teacher's mathematical studies;" "serving mathematics teachers should be in a position to keep abreast of modern developments;" "In modern society ... mathematics teachers should enjoy the esteem ... to which their ... vocation entitle(s) them;" "... the teaching profession must attract sufficient qualified teachers in this subject." I have no doubt at all that this Association endorses wholeheartedly these recommendations but it will need far more than our own efforts to achieve their operation throughout the schools of this country.

TEACHERS

No less than three-quarters of the countries involved in this survey require professional training in addition to academic competence before the individual is allowed to teach mathematics in secondary schools. Germany, Luxembourg and Portugal require a 4-year university course followed by 2 years training; in Czechoslovakia, Thailand and the Ukraine the total course is 5 years. Most countries have an organised system of refresher courses and some of these are very ambitious; in Roumania and the Ukraine teachers are required to attend a one-year course (free of cost) at the end of each 5 years of service.

About two-thirds of the countries concerned experience difficulty in recruiting sufficient qualified teachers of mathematics for the schools and a variety of reasons is given for this state of affairs. They all point to the immense technical development of recent years bringing with it increased opportunities for graduates in science and mathematics, and many of these new appointments carry greater material rewards than can be expected in teaching. In Finland, Portugal and the U.S.A. the science courses are said to be more difficult, and generally longer, than the courses in arts subjects: Czechoslovakia, Poland, Syria and Scotland instance the sheer quantitative expansion of secondary education as a main cause: Greece finds the problem much more acute in rural areas and the U.S.A. comments bluntly on the "lack of prestige" accorded to the teaching profession. Some suggestions are advanced for dealing with the situation amongst which we find: "to attempt to improve the material rewards;" "to offer special facilities to encourage students to train in mathematics;" "to re-appoint retired teachers;" "to persuade teachers now working in primary schools to transfer to the secondary field;" "to employ foreign teachers" and, in some cases, "to exempt from military service." Ceylon contemplates making mathematics a compulsory subject in

and the method of teaching, and the results were assessed by a special examination. As the report puts it: "a high percentage of insufficient marks ... will always be, for the teacher, a reason for ... surveying his work, his students, the curriculum or the testing means," and it comments that at least one of these must be at fault. Quoting as axiomatic the proposition that "in order to obtain a reasonable ... success in ... secondary mathematics, no other standard of intelligence is required than that necessary to obtain a reasonable ... success in the ... remaining school subjects," Dr. Bunt goes on to analyse the actual results and concludes that to deal adequately with the syllabus as now constructed the classical grammar schools should increase the time allotted to mathematics by 40% and the H.B.S. by 24%. Where this immense increase could come from is difficult to foresee, for out of 167 hours of instruction, which represents the total of one typical week out of each of the 5 years of the H.B.S. B course, 57 are given to their four languages, 25 to mathematics and 34 to science, leaving only 51 for all the other subjects. It is perhaps as well that it is their problem rather than ours!

CONFERENCE RECOMMENDATIONS

The delegates at Geneva produced a set of recommendations which was subsequently circulated to the Governments of all the countries taking part in the Conference. There is little in the 36 clauses of this document that is not already a normal part of the school routine in this country, and from these I select a few as illustrations: "Mathematics should be regarded as an essential part of the education of a modern person;" "Instruction in mathematics of a cultural rather than a purely technical kind should be offered as an option ... where mathematics is not a required subject;" "The difficulty and the extent of the subject matter ... should be related to the mental age ... and to the pupils' interests and needs;" "... care should be taken not to discourage the less gifted pupils by imposing on them subject matter too complex for their intellectual ability;" "to convert into skills only those processes which have been fully assimilated;" "to develop the subject, heuristically rather than to teach it dogmatically;" "to cease putting its various branches into watertight compartments;" "audio-visual aids ... play an increasingly large part in teaching, and advantage should be taken of their use to enable pupils to acquire mathematical abstractions actively."

We should, I think, agree with all these opinions and would believe that, in general, they are practised in this country. When we come to the sections concerning teachers, however, the emphasis is differently placed: "in mathematics ... more than in other

subjects ... the teacher is of prime importance;" "Teachers ... should have studied mathematics at a level considerably beyond that to which they will be required to teach;" "adequate professional training ... should be regarded as a necessary complement of a teacher's mathematical studies;" "serving mathematics teachers should be in a position to keep abreast of modern developments;" "In modern society ... mathematics teachers should enjoy the esteem ... to which their ... vocation entitle(s) them;" "... the teaching profession must attract sufficient qualified teachers in this subject." I have no doubt at all that this Association endorses wholeheartedly these recommendations but it will need far more than our own efforts to achieve their operation throughout the schools of this country.

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teacher training colleges. In this country we have already embarked upon special one-year, full-time courses to supplement the qualification of those who now hold teaching certificates but this is unlikely for some time to affect seriously the disastrous situation which is now prevalent in most of our grammar schools.

During the summer of last year I wrote to the Head of every Public School and grammar school in England and Wales, and I asked for particulars concerning the staffing and the scope of the mathematics teaching. Well over 50% of the schools favoured me with a reply and the summary of the situation makes interesting (and alarming) reading. The return from 19 of our greater Public schools shows that 149 men are teaching mathematics and of these only 20 have insufficient qualification in the subject. This ratio of 13% must be contrasted with an average of 22% from 384 county schools; of 30% from 130 voluntary schools, and of 25% from 47 Direct Grant schools. In 62 other Public schools the figure is as high as 30%; so much for the boys' schools. When we turn to the girls' schools the figures seem to be better, but the satisfaction is short-lived and in fact we know that they are in a worse plight than the boys' schools. 11 of the greater girls' Public schools show only 5 with poor qualification out of a total staff of 44, i.e. 11%, and the other figures are 16% for 174 county schools, 24% for 39 voluntary schools, 19% for 44 Direct Grant schools and 27% in 39 other Public schools. The other information which was given me related to the amount of mathematics teaching in the main school and in the Sixth Form. This shows that far less advanced level teaching in mathematics is available in the girls' schools than in the boys' schools which means that the qualified staff can be used to better purpose in the ordinary level work. But this price is bigger than we should be prepared to pay.

It is known that less than 4% of the women studying in training colleges take mathematics as one of their subjects. Even though most of the men and women working in our primary and secondary modern schools will have to teach some arithmetic, less than 45% of the women teaching in these schools have reached the G.C.E. Ordinary level in mathematics and yet, so far as mathematics is concerned, the work done in the primary schools is vital for the future development of the subject.

There is a prevalent belief that girls are in some mysterious way unable to learn mathematics. The man under whom I read applied mathematics used to say that he would rather teach seven men than one woman because "girls had never thrown stones." I am not sure that this is true to-day, but I, for one, believe that given adequate teaching the girls are not unsuited to the subject. Against some rather stern opposition I insisted on the inclusion of a clause in the

Conference Recommendations which says that "there is no reason to believe that girls are less able to study mathematics than boys."

Throughout the schools of this country teaching in English and mathematics is of paramount importance; unless these subjects receive sufficient time allocations, and the staff are both adequate in numbers and suitably qualified and trained, the standard of achievement in *all* subjects of the school curriculum must inevitable decline. Though I have some experience, I hold no brief with regard to English, but with mathematics I am closely and passionately concerned. Unless means can be found within the next few years to ensure that a sufficient number of capable teachers will enter our schools we can expect only a fall in the general standard of mathematics, a failure in the Government's plans for Technical Education and a worsening situation in the schools. There is a wide-spread belief that anyone can teach elementary mathematics. Even in primary school arithmetic teaching this proposition is dangerous; at the secondary level it is simply untrue. I hasten to pay my sincere tribute to the many women and men who are gallantly attempting to teach mathematics to a standard which is really beyond their own personal competence as teachers, but if they do not try their pupils lose even the opportunity of attempting to read the subject. The anxiety to the teacher and the risk to the pupil can be well imagined by those of us who have had the benefit of adequate training and experience.

It must not be imagined that the situation is entirely different in other countries. The Conference *rapporteur* in mathematics expressed it thus in his preliminary report: "It is necessary to find more varied ways of teaching mathematics. This would reduce one of the main causes of many set-backs: the fear of mathematics. This fear inhibits too many children and makes them accept failure before having made any endeavour. Mathematics is the subject which pupils and parents are most afraid of, ... and which is often the main cause of scholastic maladjustment."

The work of our Association and its Teaching Committee over the last 35 years has revolutionised the approach to every branch of school mathematics. No young mathematical teacher (or for that matter no old one either) need be anxious about methods of teaching. Within the Association's reports, to which will soon be added the eagerly-awaited Secondary Modern Report, will be found the collected experience and resource of generations of the best teachers of school mathematics. In their passage through the Teaching Committee the presentation and the subject matter of the reports is criticised and assessed by some of the best of our University mathematicians, so that the final recommendations can be trusted by all who use them.

CONCLUSION

The Government White Paper "Technical Education" was published in 1956, and since that time science, engineering and mathematics have been much in the news. The Times recently carried a leader on the teaching of mathematics and many other journals have published articles, in general of a highly critical nature, on the same topic. I have thought it proper, as your President this year, to refrain from any reply through correspondence columns, believing that this Address will be a better and more fitting platform for a statement about this tremendous problem. We would be the last to say that the general level of competence in mathematics by the girls and boys leaving our secondary schools was as high as it should be, and no section of the community will be prepared to take a greater share in any action which can raise this standard. Far from being complacent, we are doing everything within our power, as individual teachers and as a professional Association, to carry out the task which lies in our care. We are, perhaps, in a better position than most to make a critical estimate of the situation, and what we find disturbs us greatly.

As your President, I would speak for you and say that the standard of mathematical attainment in the best girls and boys going out from our schools will bear comparison with that of any other country in the world. I will say that the methods of teaching the various branches of our subject have received more critical attention than the methods of any other subject in the curriculum of the schools in this country. We have realised beyond question the vital importance of our subject, not only as a study with a great history of nearly 7,000 years development, but as an essential part of scientific and cultural progress. Upon the ability of this nation to make suitable expansion in this field may well depend the cultural and economic survival of our civilisation. The need is not a need for all, but for those of lesser ability who can achieve their best potential development only through the care and by the instruction of better teachers than are now working with them. We shrink at the thought of denying our best pupils the instruction of our best teachers but we do this only in the knowledge that we are ensuring beyond all question that the majority of our boys and girls will reach a standard of achievement lower than is necessary and desirable.

It is, in my view, useless to say "we must double the number of technologists by 1970." As things stand at present neither the teachers nor the suitable student material is available to accomplish this task. It is admitted that if we dared to distort the curriculum of our schools in such a way that little more than English, mathematics and science was taught after (say) the age of 13 we could

accomplish the task with something to spare—and this something, that we can very easily spare, would be the death warrant of our British way of life. Rather must we face the problem from the other end. With the cooperation of the schools it should be an easy matter to know that in 2 years time rough totals of x , y and z boys and girls will be coming out of our Sixth Forms capable of reading mathematics in an honours course, mathematics as a subject in a general degree, and mathematics to a standard required by the special subject courses in the teaching certificate. It can then be seen what provision our Universities, Technical Colleges and Training Colleges can make for these young people, for I believe that we dare not extend in any unlimited fashion the size of the mathematics faculties in our universities; to lower the general standard of the faculty will in the end drag down the level of the best. Once the further education of these students has been assured (and this may mean wholesale revision of such things as maintenance allowances for Sixth Form courses, the extension of the provision of county major awards and similar material necessities) we must then investigate the use to which the country will put the qualified graduates and teachers that she has available. We must control the appointments which are available, not dream of directing individuals into particular posts. Far too many men and women are now going into appointments in which their special qualifications are not being used to full advantage, and with the number at our disposal in this country we can ill afford what really amounts to a gross waste of academic power.

The hardest task of all will be to ensure that sufficient capable, qualified and willing people are attracted into the teaching profession. Russia claims that planning to meet estimated needs has solved the problem of teacher recruitment. In 1955 no less than 80% of the graduates from the Universities were directed into teaching. In this country any attempt at direction or compulsion would produce a state of affairs worse than that which we know at present, for beyond his professional competence the teacher must choose his life's work because he wants to teach, and then be prepared to equip himself adequately for the task which is, so far as I can judge it, one of the happiest, most valuable and most rewarding of all vocations. But this task must be accomplished; the debt that we owe to those who launched this great revolution in the past demands that we shall face the challenge of our time, and of the future, and keep the "Queen of Sciences" secure upon her throne.

W. J. L.

ANNUAL MEETING 1958

The Annual Meeting of the Association was held in Manchester from Wednesday, April 9th to Saturday April 12th. The meeting was very well attended. The President Mr. W. J. Langford was in the Chair.

The following elections to Honorary Memberships were made by Council.

William Charles Fletcher. C.B., M.A.

William Hope-Jones. B.A.

Charles Orpen Tuckey. M.A.

George Neville Watson. F.R.S.

In reporting the elections the President recalled the services which the new Honorary Members had given to their subject and to the Association.

Mr. W. C. Fletcher was a Fellow of St. John's College, Cambridge and Second Wrangler in 1886, and H. M. Chief Inspector of Secondary Schools 1904-1926. In 1909 he drafted the famous Ministry of Education Circular 711 on the Teaching of Geometry and in 1912 the 2 Volume Report on the Teaching of Mathematics. He was President in 1939 and held office until 1944.

Mr. W. Hope-Jones was a Scholar of Eton and Exhibitioner of King's College, Cambridge, and a Wrangler in 1906. He returned to Eton in 1907 where he remained for the next 43 years. He has been a member of the Association since 1917 and was President in 1938.

Mr. C. O. Tuckey, was a Major Scholar of Trinity College, Cambridge, and a Wrangler. After four years teaching at Winchester he became Senior Mathematics Master at Charterhouse where he remained for 37 years. He played at Wimbledon and is the husband of, and father of, Wimbledon championship winners. He joined the Association in 1902 and has been a member of the Teaching Committee ever since, and Chairman of the Committee for a great many years. He was President in 1944.

Dr. G. N. Watson was Senior Wrangler in 1907 and Professor of Mathematics in the University of Birmingham from 1918 to 1951. He was awarded the Sylvester Medal of the Royal Society in 1946 and the Dr. Morgan Medal of the London Mathematical Society in 1947. He has been a member of the Association since 1913 and a regular contributor to the Gazette for over 50 years. He was President in 1932 and 1933.

Mr. A. W. Siddons was a pupil of Rawdon Levett at King Edward's School, Birmingham, Scholar and Fellow of Jesus College, Cambridge, Wrangler in 1898, and Mathematics Master, and Senior Mathematics Master at Harrow for nearly 41 years. He

was the first Secretary of the Teaching Committee, from 1902 to 1913 and President in 1935 and 1936.

After the Presidential Address Prof. M. J. Lighthill, deputising at short notice for Professor Behnke, who was unable to be present, spoke on "The Teaching of Applied Mathematics in School and University." Admitting his lack of experience in school teaching, he nevertheless proposed three most valuable principles which should be kept in mind. (1) In all school Mathematics teaching, boys and girls should be trained to apply their Mathematics to life, and not to regard it as a piece of ceremonial ritual. This applied equally to Arithmetic and Algebra as to the more obvious application of Mechanics. (2) Before Mathematics could be applied to any subject, that subject must be understood practically. The physical sciences took pride of place in school work, therefore there were overwhelming arguments for taking Mechanics as the first thorough application of Mathematics: the Statistics required for biology had wide application, but was probably too difficult for immature students. (3) Ideas, and the proper expression of them, were equally important. Boys and girls must be taught to write good English—even to correct punctuation—and to express themselves in simple, lucid, style. In conclusion the Professor outlined the courses in Applied Mathematics at Manchester University, in which the emphasis on practical value as distinct from analytical interest was apparent.

Mr. C. Mack of the Shirley Institute, read a paper on "The Application of Mathematics to Industry." He discussed a variety of problems, including the timing of processes of production, linear programming and queuing. To exemplify research in Applied Mathematics he took the problems of friction in textiles, where in many cases the frictional force had to be taken proportional to a fractional power of the normal reaction, and the problem of "ballooning" in spool winding, which involved Bessel functions.

In the discussion of the Association's Algebra Report, Prof. M. H. A. Newman emphasised the modern recognition of algebra as a subject in its own right—the domain of finite operations—and not merely as the handmaid of analysis. The standard of rigour to be aimed at in school algebra was that of "good proofs with gaps:" gaps that the average schoolboy was not likely to suspect. It was a mistake to discuss difficulties that had not been appreciated by pupils. Special topics for which he put in a plea were: simultaneous linear equations not equal in number to the unknowns; the Theory of Numbers, including Fermat's theorem; Finite Differences, as far as Newton's interpolation formula; Complex numbers, treated as numbers and not as points or vectors. Miss Batty declared her appreciation of the Report, with special reference

to the sections on the nature of proof and the distinction between reversible and irreversible implication, partial fractions, and inequalities. She felt that the method of "first failure" was harder than the classical method of mathematical induction, and that the arrow symbol for implication was a valuable suggestion, useful with the better pupils. Several members contributed to the discussion that followed, most of whom commended the report warmly. Many commented on the chapter on Partial Fractions; some remained unconvinced by the assurance of others that it was never necessary and almost always quicker not to use the method of undetermined coefficients, the abolition of which in connexion with partial fractions was recommended in the report. The chief critic of the Report was Mr. Donnellan, who felt that an opportunity had been missed of rethinking school algebra in the light of modern developments in linear algebra and group theory, and that too subservient an attitude was still adopted towards examination syllabi.

Dr. C. B. Hazelgrove gave a lecture on "Computing" with special reference to the work done on the electronic computers at Manchester University. He explained a simple example of programming for the evaluation of a polynomial function, and described various uses of the machine; weather changes could now be forecast before they actually occurred! Calculations which took six months on EDSAC I at Cambridge could be checked by MERCURY in a single night. He discussed the possibility of doing real mathematics with a computer, as distinct from numerical computation. Various problems in number theory had been investigated; Polya's conjecture with regard to the ζ -function had been disproved; useful numerical work on the zeros of the Riemann-zeta function had been carried out, and certain constants in the theory of approximation to irrationals had been discovered. He himself felt that machines could be programmed to do work in logic and algebra which would simplify "real mathematics."

Dr. P. L. Taylor's subject was "Mathematics for Industry" and was enlivened by several humorous references to the foibles of mathematicians and engineers. By means of a simple example in automatic control he showed how the formulation of a problem in mathematical terms and the subsequent interpretation of its solution were at least as important as the mathematics itself, which the engineer regarded as merely one of many tools which he could use to get practical results. He stressed the need in teaching Mathematics to teach not only technique but also the philosophy of the subject—how the mind of the mathematician "ticks."

Dr. R. D. Davies described the work at Jodrell Bank which was making it possible to plot the distribution of neutral hydrogen in

the universe by measurement of its emission of radio waves in the region of 1420 Mc/s. The presence of considerable masses of hydrogen had been long suspected from considerations of dark clouds and estimates of the mass of the galaxy; it was now proved by direct experiment. Displacement of the fundamental radio-frequency by the Doppler effect enabled us to measure the motion of various parts of the galaxy and threw a flood of light on its structure. Radio reception could also teach us a great deal about the evolution of the stellar universe and might ultimately enable us to decide between the discontinuous and the steady-state theories of the origin of matter. There was also a plan to use the large radio-telescope as a radar transmitter, and in particular to obtain echoes from the planet Venus, which should help us to elucidate the surface conditions obtaining there.

The meeting concluded with a light hearted discussion "That Geometry ought to be abolished."

CORRESPONDENCE

ANGLES AND NUMBERS

To the Editor of the *Mathematical Gazette*

DEAR SIR,

All teachers will surely applaud Mr. Hope-Jones' vigorous and amusing exposure of the disastrous consequences of over-emphasis on the degree as a unit of angle. (If anyone would like a simple confirmation of this, let him try marking the angles of a 30° , 60° , 90° triangle $\frac{1}{6}\pi$, $\frac{1}{3}\pi$, $\frac{1}{2}\pi$ and observe the sensation it causes). The error to which he draws attention, however, is also due to an equally serious misdirection of emphasis at a higher level.

Think, first, of all the calculus books you know which have sections on differentiating $\sin x^\circ$, and then consider the harm these may have done. No mathematician endeavours to differentiate t seconds, or x^2 cm², for the very good reason that such a process would be as impossible to justify logically as is the algebra error of ' $s = 10$ miles.' Yet a degree is an arbitrary unit of a similar type, and it is difficult to see that there is sufficient difference between these examples to make the attention given to differentiating $f(x^\circ)$ desirable. Instead of merely stressing that the angle must be measured in radians before any attempt is made to differentiate a circular function and leaving it at that, these unnecessary and possibly misleading formulae have been perpetuated by successive textbook rehashers.

The radian (or, for that matter, the straight angle) is, of course, a *natural* unit of angle in a sense in which no unit of time or length can ever be said to be natural. But the main reason why such natural units are *possible* for angles and not for most other quantities is seldom made clear to students. It is merely that angle is a dimensionless quantity, and teachers will be wise to drive this fact home often. The old habit (mercifully obsolescent) of adding a stupid little ° to indicate radian measure also helped to conceal this important feature.

The real problem, however, lies much deeper than this. From the point of view of higher mathematics, it is most unfortunate, even if inevitable, that pupils first meet the circular functions as functions of an *angle* rather than a *number*, and that this happens when they are at a very impressionable age. The real reason that these functions are important in mathematics is simply that they are periodic functions of their argument. But some students never seem able to forget that they first met sine as 'opposite over hypotenuse.' I have occasionally been astonished to find that this attitude has survived even among scientists capable of taking S Level and meeting, say, $\text{cis } \theta = \exp(i\theta)$ for the first time. Close investigation has sometimes revealed that a boy has been puzzled by some such formula, not because of difficulties with the complex numbers as such, but because he has **ONLY** been able to think of the θ on the left side as an *angle*. If one tries to define $\cos(a + ib)$ with such a person, he will immediately begin worrying about how $(a + ib)$ can be an angle, even though he may be reasonably happy about other complex functions, such as $\text{ch}(a + ib)$. This means that during all the years in which he has been writing $\sin^{-1} x$ for $\int (1 - x^2)^{-1/2} dx$, he has never properly grasped the fact that these functions have exactly the same numerical status as the $\ln|x|$ he has been obtaining for $\int x^{-1} dx$. It is an alarming commentary on the success of one's teaching, I can assure you.

The solution obviously lies in taking every opportunity to speak of the tangent of a *number*, rather than the tangent of an angle, and so on. The very *first* time the radian is discussed, it should be pointed out that, with this unit, $\sin x$ becomes a function of the *number* x , just as much as x^3 or \sqrt{x} . This dogma should be re-affirmed before any attempt is made to differentiate any circular function, again when the inverse functions are introduced, again before obtaining the series expansions for $\sin x$ and $\cos x$, and so on. One might even venture to suggest that manufacturers of trigonometric equations should sometimes ask their victims to find all the *numbers* between 0 and 2π which are solutions. $\sin^{-1} x$ should sometimes be read as 'the number whose sine is x ' and *never* as 'the angle whose sine is x .' If Mr Hope-Jones' candidate had

received this sort of training, he would have been less likely to have evaluated his integral in square degrees!

Incidentally, I should like to take this opportunity to disagree profoundly with another writer in the same issue of the *Gazette* who regrets the British use of \sin^{-1} and prefers the continental \arcsin for these inverse functions. While I agree that there are a few purposes for which $\sin^{-1} x$ is not an ideal notation, I think that $\arcsin x$ is a far worse choice, (quite apart from its clumsiness and the fact that $\arcsin x$ by analogy seems rather absurd). Remember that it says 'the *arc* whose sine is x ,' and though this approach has a considerable historic interest, it should be clear from the arguments above that by the time the modern pupil has gone far enough to require these inverse functions, he should not be thinking of the sine of an arc any more than of the sine of an angle, but only of the sine of a number. While it is true that weaker pupils are often bewildered by \sin^{-1} , I cannot agree that, from a more farsighted viewpoint, our notation is as 'misleading' as is claimed, even though an appreciation of its subtlety does perhaps require a greater mathematical maturity than can be expected from the majority of schoolboys. The only satisfactory justification is to present \sin as an operator, with inverse \sin^{-1} . These operators satisfy the identities $\sin \sin^{-1} x \equiv x$ (for $-1 \leq x \leq 1$) and $\sin^{-1} \sin x \equiv x$ (for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$). Such manipulations ($TT^{-1}x \equiv T^{-1}Tx \equiv x$) of transformations, mappings, permutations, automorphisms, etc. are, however, commonplace in modern algebra, and it probably won't be too many years before this flourishing subject has penetrated the school curriculum. Our maligned \tan^{-1} notation, therefore, may eventually prove to be of considerable anticipatory value to our students. (In the meantime, the most urgent reform needed at the school level is a *consistent* policy by examiners to reserve the symbol with the lower case initial letter exclusively for the principal value of the inverse function, and not to use it indiscriminately for either the principal or the general value, as happens with at least one Examining Board.)

For a similar reason, I cannot share the same writer's enthusiasm for the continental and American habit of deliberately confusing an arc length with an angle, however many examples history may provide to sanction this procedure. (There should be no dismay at this; many interesting early treatments, e.g. of series or of complex numbers, are considered inadequate by modern standards.) Having recently had the opportunity of teaching geometry according to this scheme, and having been obliged as a result to tell frequent lies to save contradicting the textbook *too* often, I challenge the contention that it gives 'clearer insight and a valuable generalization.' No confusion between quantities of different mathematical significance

(and in this case of different dimensions, even) can ever be considered to have clarified the presentation. In Britain, it would only accumulate a wealth of trouble for future work in mechanics and differential geometry, where our syllabus requires an easy familiarity with transformations of the type $s = a\theta$, $v = \dot{s} = a\dot{\theta}$, etc. The formulation this writer wishes us to adopt is, in fact, almost as pernicious as the definitions of the trigonometric functions used by some American writers, who draw a circle of unit radius and *define* their circular functions as *lengths*!

Yours etc., ROGER F. WHEELER

Hymers College, Hull

THE DEFINITION OF A LOCUS

To the Editor of the *Mathematical Gazette*

DEAR SIR,

Mr. Wheeler's criticism (*Gazette*, XLII, 61) of the too-prevalent kinematical account of loci (not 'dynamic,' surely, even in Clifford's idiom) is well directed, and it can only strengthen his case to protest that his definition of a locus has been orthodox for a very long time.

It is not quite the original definition, for in the Greek the stress when the new word was introduced seems to have been on the theorem, not on the set of points identified in the theorem. The locus classicus, so to speak, is a sentence in the *Commentaries* of Proclus. The first English translation, by Thomas Taylor in 1792, runs (Vol. II, 177) "I call those (theorems) local, to which the same symptom happens in a certain place." Few of us would interpret this sentence with any confidence, even with the help of an entry in Thomas Walter's *Mathematical Dictionary*, 1762; "Local problem, such a problem as is capable of an infinite number of solutions, and all different." But Heath in his *Euclid*, 1908 (vol. I, 329) gives us an intelligible version, "I call those (theorems) locus-theorems in which the same property is found to exist on the whole of some locus," thus claiming in effect that the idea of a locus as a propertied class is classical.

This is not to suggest that this idea had the same generality long ago as it has to-day. Until recently there has always been a tacit assumption that only relations of certain kinds were recognized in polite society. To L'Hôpital and Maclaurin in the first half of the 18th century, for example, dazzled by the invention of coordinates, a locus is the locus of an equation. Again, in the sentence "To every property in relation to each other which points can have, there corresponds some locus, which consists of all the points possessing the property," A. Whitehead in his *Introduction to Mathematics*, 1911 (p. 121), seems to be making no reservations, and it is a shock to find him in the preceding sentence asserting

that a (plane) locus is a *curve*. Is not $AO^2 + BO^2 < \frac{1}{2}AB^2$ a property of the variable point O in relation to the fixed points A, B ?

In the Association's Geometry Report of 1923 the emphasis is different, and the writers take for granted that they are using familiar language in saying (p. 63) "The locus that corresponds to a prescribed set of conditions is both inclusive and exclusive, including every point which satisfies the requirements, excluding every point which does not." The Report gives examples of loci which can not be specified by equations or described by moving points. "Such loci enliven the class-room."

Teachers' blunders necessarily tend to perpetuate themselves, and I hope Mr. Wheeler will continue his campaign. The *Encyclopaedia Britannica* and the *Concise Oxford Dictionary* can be quoted against him, but there is some ground for hope. It is by way of crosswords that most of us extend our vocabulary nowadays, and *Chambers* must have the last word. "Locus: (*math.*) the line or surface constituted by all positions of a point or line satisfying a given condition."

Yours etc., E. H. NEVILLE

UNSOUND EXAMINATION QUESTIONS

To the Editor of the *Mathematical Gazette*

DEAR SIR,

Until 1937, "The Mathematical Gazette" had a feature called "The Pillory" in which unsound mathematical questions set in public examinations were shown. Then, a member who found such a problem knew what to do. He simply sent the question to the Editor, with or without comment. Now, as then, he may first ask the problem bureau for a solution, but it is not clear what is to be done next.

The Joint Four Secondary School associations send an annual questionnaire on G.C.E. exams to schools, and any criticisms of the papers, including accusations of unsoundness, are best made in this way. In College Entrance Scholarship examinations, the proportion of unsound questions is greater than in G.C.E., and the colleges may receive a stream of letters about these mistakes. It would be helpful if there were some way in which *one* official protest could be made against every unsound question.

No one knows better than the examiner who has tried to clear up the mess after a dud question, that such things should not reach the candidates. The examiner himself must not be blamed, any more than the printer. The examining body should organise itself so as to minimise the chance of an unsound question's being overlooked. Examining bodies whose record is bad should try to

get advice from those who are more reliable. It is not a question of mathematics, but one of statistics and psychology.

It might be interesting to attempt a classification of unsoundness, from the obvious misprint to the hopelessly obscure question, but it would be difficult. To conclude, here are three examples of rather unusual types: An ambiguous question, set to candidates for Science scholarships: A uniform rod of length $2a$ hangs, in a horizontal position by two light vertical strings of length b . Find the period of small oscillations.

A well-known result that is difficult to prove, also set to candidates for Science scholarships:

Prove that the solid which has the greatest volume for a given surface area is a sphere.

Set in London Inter. (Eng.), a question with too much data: This contained seven incompatible measurements, any six of which would give an answer, so that the seven answers were different. Other "correct" answers could be obtained by using all the data in various ways. This sort of question does not worry a candidate unless he has time to check his results, but is interesting to mark.

Yours etc., G. A. GARREAU

To the Editor of the *Mathematical Gazette*

DEAR SIR,

I should like to suggest two other windmills for Mr. Hope-Jones to tilt at. No doubt if he were fined 40 shillings he would indignantly defend the rights of the pound sterling; but how much more serious a matter might it be if the fine were mistakenly entered as '2 lb.'

Those of us who have to teach our pupils to distinguish between mass and weight, and who try to restrain them from using such phrases as *a force of 3 lb.*, receive too little support from some examiners and some text-book writers.

Many of us, again, teach our pupils to use letters to represent numbers, rather than distances, times or sums of money. We make them start an algebra problem, for example, by writing *Let x miles be the distance*, and finish it with $x = 3$. *Therefore the distance is 3 miles.* We hope that, trained in this way, they will not go wrong when they meet such a statement as *At h feet above sea level the distance of the horizon is approximately $\sqrt{(3h/2)}$ miles.* But can we expect them to be more particular about these points than the examiners whose questions they have to answer?

There is, too, the unsatisfactory compromise *Let x be the distance in miles*, inaccurate and misleading, but tolerated because it is customary. Either x is a number or it is a distance: it cannot be both at once.

Yours etc., E. H. LOCKWOOD

To the Editor of the *Mathematical Gazette*

DEAR SIR,

I read the Gleaning No. 1904 in the last number of the *Mathematical Gazette* with some concern. As a mathematician and amateur sailor I can assure you that the Daily Herald is right provided the expression "much faster" is interpreted not too stringently.

Yours etc., H. HEILBRONN

ALAN ROBSON

PRESIDENT OF THE MATHEMATICAL ASSOCIATION, 1949.

The death of Alan Robson in 1956 deprived the Mathematical Association of services whose value only those who were in close contact with him can appreciate adequately since much of his work was done behind the scenes.

He served on most of the sub-committees which have been responsible for the major reports issued by the Association during the last thirty years, through which indeed the influence of the Association on teaching methods is largely exercised, and he took an active part in their composition and preparation:

Mechanics (1929); Arithmetic (1932); Algebra (1934); Geometry, second report, (1938); Trigonometry (1950); Calculus (1951); Algebra in Sixth forms (1957); and the succession of book-lists for School Libraries (1936, 1945, 1954).

Mention should also be made of an important meeting held in Cambridge in 1937 under Robson's chairmanship to discuss the extension to sixth form work of the reports of the Association which up to that time had hardly gone beyond the range of the School Certificate Syllabus. Plans were made but their execution was delayed by the War. It should however be put on record that with a few others Robson prepared unofficially a draft report which formed the basis of the report on *Higher Geometry in Schools*, (1953). Robson did not serve on this sub-committee because he was fully occupied at that time with the preparation of the Calculus report as its editor but advantage was taken of his wisdom and experience by informal consultation when difficulties, inevitable in a pioneer report, arose in its preparation.

Robson's work for mathematical reform extended far beyond this committee work. He belonged to the first generation of school-masters who enjoyed the fruit of the renaissance of mathematics at Cambridge for which Hardy, Russell, Bromwich, Baker, Hobson and others were responsible. There are now few who can realise

how great was the change in the mathematical atmosphere of Cambridge before and after the abolition of order of merit in the Mathematical Tripos, itself a symptom rather than a cause of reform. But these changes in the teaching of analysis and geometry at the University exerted in their turn a growing and deepening influence on the training of school specialists, as no doubt generations of Marlburians who studied under Robson are especially aware.

Apart from insisting on the accurate use of language and logical statement, perhaps the most interesting of his qualities was a passion for economy in material and presentation which showed itself in the stress he placed on the structure of mathematics rather than on applications. His contributions to teaching method belong to the same category as those of Sir Percy Nunn who has influenced profoundly main-school mathematics both by the originality and versatility of his books and by his work at the London Day Training College. Robson however was more interested in Sixth Form work and it is probable that his influence was exerted most effectively through the lectures on post-certificate work he gave for the last thirty years of his life to teachers who attended the summer courses arranged by the Board of Education. All who heard for example his talks on economy of material in proofs of algebraic inequalities and on the nature of abstract geometry could not fail to be stimulated by the point of view he presented.

Some sixty years ago, by a happy inspiration, Greenstreet adopted as his text for the Gazette:

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and an ornament thereunto."

To this appeal, the responses made by Nunn and Robson were alike whole-hearted. Could there be any finer tribute than this to two great teachers?

C. V. D.

"THE TEACHING OF ALGEBRA IN SIXTH FORMS"*

BY M. H. A. NEWMAN

It is just on twenty years since I was first permitted to air in public some of my views on school algebra. That talk, in 1937, was officially part of a British Association discussion, but since the other speakers were yourself, Mr. President, Professor Broadbent and Mr. G. L. Parsons, with Professor Neville in the chair, it was quite a Mathematical Association family party. That was also the occasion of my becoming well acquainted with Alan Robson, who during the intervening years did so much, by his splendid text-books and by the influence of his pupils, to bring new ideas into school mathematics, and particularly into school algebra. It was a great pleasure to meet and correspond with him after so long an interval, during the years in which the Report that we are to discuss to-day was in preparation. His sudden death at the very time when the Report was first presented to the Teaching Committee of the Association was a most sad and unexpected blow. His thoughts had been occupied a great deal with it after his retirement, and it owes to him not only the chapters for which he was mainly responsible, but innumerable improvements at all the points that his sharp critical eye had lighted upon.

On looking again at my 1937 talk I find that although the years may have made me a shade less sanguine about the range, or, as I believe it should now be called, the band-width of schoolboy reception, what I then put forward as my main thesis has had some influence on the present Report. This was the thesis that algebra is not to be regarded, even in school work, as a collection of oddments left over from other branches, but as a well-defined and unified subject particularly well suited for giving school pupils a taste of rigorous argument comfortably surrounded by the jam of formal manipulative work. It is for this reason that the present Report has kept strictly, in its main chapters, to what is properly called Algebra, that is the study of the four rules of addition, subtraction, multiplication and division, and certain other finite processes, and has excluded matters, such as limits and infinite series, which are still for historical reasons often included in treatments of the subject. If this distinction can be broadly maintained in teaching, even though nothing is said explicitly about it, pupils may be given an impression of simplicity and purity in the algebraical argument which is not attainable in analysis at the level of school mathematics. It was decided at the outset that the Report would not attempt to deal with the balance and composition of school

* The opening contribution to a discussion of the Associations recently published Report, at the Annual Meeting of the Association in April 1958.

syllabuses as a whole, which must vary so much with circumstances that generalisations are almost useless, but should be about the mathematical materials and their treatment. We have made suggestions about new topics, and new ways of teaching old topics, which teachers may like to try out in practice when the opportunity arises; and sometimes, since the methods are new, we have given some arguments in their favour. But this does not mean that in our view they are necessarily to be preferred to the old methods and subjects, but only that we think the new ones worth a trial.

I will refer briefly to some of the topics not generally included in school courses at present, to which attention is drawn in the Report. A rather thorough treatment of simultaneous equations in two or three variables, when the number of equations is either greater or less than the number of unknowns, or when the determinant vanishes, is a very good way of implanting in pupils a first suspicion that there may be interesting general ideas behind the rules they learn for solving equations; and the geometrical interpretation in terms of intersection of planes and lines throws light on both subjects. Whether this piece of work can often successfully be carried further, to the ideas of linear independence and the algebra of matrices of small order, which was the gospel I preached in 1937, I now feel a little more doubtful. Robson, whom I managed to fire with enthusiasm for the idea at that time, was successful in teaching this theory to many generations of his Marlborough pupils, and remained a convinced supporter of it as a school topic. Certainly, if it is feasible, it is a valuable and educative piece of work, and it would be useful if it could be given a more wide-spread trial.

The theory of numbers has still a somewhat precarious place in the school course. If a sufficiently substantial connected piece is done to show that it is a subject and not a collection of curios it has particularly strong claims to be included. Both the proof of Fermat's theorem and the ingenious uses that can be made of it to prove, for example, congruences between large numbers, strike an entirely new note in school mathematics. The similarity between the H.C.F. and factorisation properties of numbers and of polynomials can be the theme of explanations that go as deep as the teacher thinks his pupils' mathematical lungs will stand. The history of mathematics, as something which did not drop from the sky complete with sets of examples, can very easily be touched upon in connection with the Theory of Numbers. Euclid's algorithm and Euclid's proof that there are an infinity of primes give a chance to say something about a great mathematician whom the proper zeal for freedom of the geometry reformers 50 years ago has banished too completely from the ken of school children. A final reason not to

be ignored for introducing pupils to the theory of numbers is that it is, of all subjects taught at school, the one that gives the best chance, to people who have mathematical tastes and ability, but no opportunity for further study, of keeping up an amateur interest in mathematics in later life.

A course on the early stages of the numerical methods of finite differences, up to Newton's interpolation formula, should be well within the powers of Sixth Form mathematicians. Such a course would have the practical advantage of providing schoolboys and girls with a sample by which to judge in good time whether or not they would like to take up any of the many present-day occupations in which systematic numerical work plays an important part.

On the suggestions for alternative treatments of the customary topics it would be useless for me to speak in detail. They have been put out to the best of our ability in the Report and will no doubt be commented on, I hope not too severely in the rest of the discussion. I can more usefully refer to some general principles which have been stressed at a number of points in the Report.

In your contribution to that British Association symposium, Mr. President, one of your themes was that school teaching should not be allowed to become too academic. I was glad to notice some unmistakable signs, in your address yesterday, that you felt as strongly as ever on this point. It is a view which finds expression early in the Report in the principle, laid down at the beginning of the introduction, that the standard of proof required in school work should not be the full rigour of university mathematics. I have never found anyone to oppose this principle, but it seems nevertheless to awaken some misgivings when it comes to the applications. Our interpretation of it in writing this Report was, that although we will not accept proofs that are definitely faulty, or which are not only incomplete, but cannot be completed, we will accept, and teach, good proofs with gaps. This means that we are willing to pass over in silence steps that seem obvious, although at a more exacting level they may demand careful justification. "In silence" is the point. That exhortation from France that you read to us yesterday, Sir, "clearly and honestly to indicate the places where assumptions are made without the support of the necessary proof," would if followed bring more glory to the teacher than enlightenment to the pupils. It is nearly always harder to explain that there is something to prove at one of these places, than to carry out the proof. The proper course is to glide smoothly past. A great deal of this kind of beneficent cheating is inevitable. The suggestion that is made, at several points of the Report, is that the knowledge that a few such omissions will be made should be used, to choose deliberately treatments which thereby become

simple and intelligible. A typical example is the treatment of complex numbers which, for many years now, have usually been taught on the basis of the rather abstract notion of "number-pairs": one complex number is a pair of real numbers. Somewhere in any rigorous treatment the number-pairs, or some equivalent model, must have their place; but the axiomatic methods which have assumed such a dominant position in contemporary work in algebra, have made it clear that the real purpose of the number pairs is to provide a formal proof that the theory of complex numbers is *consistent* (or at least as consistent as the theory of real numbers). Now this is the very step which the schoolboy can surely be spared. An informal explanation of why — I can (after all) have a square root, so much he must have; but formal consistency proofs, I suspect, he is willing to do without. When teachers say, as one or two do, that there is really no difficulty about number-pairs once you get used to the idea, I wonder who it is that has got used to it. I cannot help recalling on these occasions Professor Littlewood's remark years ago that after giving the same course many years running he could never help feeling that really, after all this time, they ought to know it. An outline is given in the Report of a method of introducing complex numbers in a way which some, at least, may think more direct, and which is yet quite proper if the doctrine of the limited liability of school proofs is frankly accepted.

Since not only the treatment of complex numbers but also parts of the chapters on the theory of numbers and of polynomials are closely related to axiomatic notions which are only now finding their way into the undergraduate syllabus in many English universities, we thought it would be useful to give a very brief introduction to these notions in an Appendix to the Report. Although we have done our best to emphasize that these matters are for the teacher's own background, it is difficult to make sure that there is no misunderstanding on this point. It is to be feared that some will think we are proposing that school algebra should henceforth be rigorously developed from a stated set of axioms; and then we shall be hot on the trail of those countries you spoke of yesterday, Sir, whose lofty aims leave us so far behind. Indeed our aim has been just the opposite, to resist premature academicism by providing, *for the teacher*, the means of convincing himself that he is justified in using some simpler and older methods of introducing the concepts of algebra, avoiding complicated constructions; and also in separating algebra from geometry and analysis when he wishes to. The interrelation between the three subjects is indeed important and illuminating, but not so important, I think, as a clear view of each of them, including a firm conviction that numbers are things you add and multiply. On the evidence to be gained from

university entrance candidates, I believe that the effect of learning at school that complex numbers are points or vectors, or that one number is two other numbers, is not enlightenment but a sinking feeling. The aim of this Report in so far as it deals with such topics is to support the use in school work of sound arguments within the agreed limitations, based upon simple and natural concepts.

M. H. A. N.

COLLOQUIUM FOR TEACHERS OF MATHEMATICS IN SCHOOLS

Held at Downing College, Cambridge, 29th July to 2nd
August, 1957.

This colloquium, organized by the Faculty of Mathematics and the Board of Extra-mural Studies on the same lines as that held in the summer of 1956, offered teachers an opportunity to re-establish contact with University mathematics, and attracted a large number of applicants for its sixty places.

Three lectures by Dr. Zeeman on Abstract Algebra introduced the ideas of groups, rings and fields. Mr. Lindley sketched the foundations on which statistical methods are built and the purposes for which they are used, and followed this by indicating the scope of the University courses in Statistics. Professor Bondi gave two persuasive lectures on Newtonian Cosmology; Dr. Wilkes and Dr. Miller one each on Electronic Computers and Numerical Analysis.

Afternoon visits were arranged to the Observatory, the Engineering Laboratories, and the Mathematical Laboratory. These were full of interest and contained only one disappointment when EDSAC II, to whose performance we had looked forward, got stage fright and blew her main fuse.

Six evening discussions were organized. Four of these concerned the school teaching of subjects which come on the fringe of the syllabus: Analysis, Probability and Statistics, Projective Geometry (now treated in Part I of the Mathematical Tripos by means of matrix algebra); and the possibility of introducing Numerical Analysis. Other subjects discussed were Careers for Mathematicians, and Mathematics as an introduction to Engineering. Each member of the conference could attend only two discussions; this was a regrettable, though probably necessary, limitation.

Our thanks are due to all those who made the colloquium such a success. It was an encouraging and stimulating experience, and one hopes that those who were turned away this time will have their chance in 1958—including perhaps a view of a sputnik and a proof of Fermat's last theorem.

A. BARTON

CLASS ROOM NOTES

21. A rugby-field problem

"In taking a place-kick to convert a try into a goal, the ball may be placed anywhere on a line drawn at right angles to the goal line from the point of touch-down. Is there a point from which the chances of conversion are greatest, assuming no limit to the kicker's powers?"

This problem may be treated at three levels; as a scale drawing problem for Juniors, as a problem in Pure Geometry at *O* level, and as a problem in Calculus at *A* level.

Let *A*, *B* be the goal posts a distance *c* apart, and let the point of touch-down lie on a line *l* perpendicular to *AB*, at a distance *a* from the nearer post. Finally let the circle through *A*, *B* touching *l*, touch at *P*. Then *P* is the point on *l* at which *AB* subtends the largest angle. For if *Q* is any other point on *l*, so that *Q* lies outside the circle and if *QA* meets the circle again at *P'* then angle *APB* = angle *AP'B* > angle *AQB*. If the distance of *Q* from the goal line is *x*, by considering the angles *l* makes with *AQ* and *BQ* we may express the connection between *x* and angle *AQB* = *θ* in the form

$$\begin{aligned}\cot \theta &= \left(1 + \frac{a(a+c)}{x^2} \right) \bigg/ \left(\frac{a+c}{x} - \frac{a}{x} \right) \\ &= \frac{x}{c} + \frac{a(a+c)}{cx}\end{aligned}$$

whence
$$-\operatorname{cosec}^2 \theta \frac{d\theta}{dx} = \frac{1}{c} - \frac{a(a+c)}{cx^2}$$

so that $d\theta/dx = 0$ when $x^2 = a(a+c)$, and so *Q* is the point where the circle through *A*, *B* touches *l*. Since $d\theta/dx$ changes from positive to negative as *x* increases through the value $\sqrt{a(a+c)}$ this value of *x* gives the maximum value of *θ*.

Chingford County High School, E. 4.

H. T. FISHER

22. Comment on class room note No. 1

Young students who have been wrestling with quadratic equations by completing the square always react with pleasure not unmixed with incredulity to Mr. Hesselgreaves' suggested method of solution by merely drawing a circle. The method can be slightly generalised by taking (for $x^2 - px + q = 0$) the diameter not necessarily from (0, 1) to (p, q) but from (0, a) to (p, b) where $ab = q$. This gives a whole set of coaxial circles, any one of which gives the roots.

Ilminster, Somerset.

W. H. COZENS

23. The remainder theorem

The usual school demonstration of the Remainder Theorem consists of either (1) performing the actual "long division" of some typical polynomial such as $ax^3 + bx^2 + cx + d$ by $x - \alpha$, a device which laboriously churns out a quotient which is of no immediate interest; or (2) more abstractly, asserting the identity $P(x) \equiv (x - \alpha) \cdot Q(x) + R$, and then setting x equal to α , a step which causes uneasiness in the minds of pupils (and teachers) if they get as far as (and no further than) questioning the validity of division by a divisor which becomes zero as soon as its work is done.

Here is another demonstration.

We recall that to divide is to express a number as a multiple of the divisor. In a simple case like $45 \div 7$ it amounts to rewriting "4 tens + 5" as "6 sevens + 3". More generally, to divide the polynomial $f(x) \equiv a + bx + cx^2 + dx^3 + \dots$ by $x - \alpha$ is to rewrite it as a sum of multiples of powers of $x - \alpha$. To accomplish this transformation, set $x - \alpha = y$, i.e. $x = \alpha + y$. Hence $f(x) \equiv a + b(\alpha + y) + c(\alpha + y)^2 + d(\alpha + y)^3 + \dots$. The first term of each bracket yields in sum an absolute term

$$a + b\alpha + c\alpha^2 + d\alpha^3 + \dots \equiv f(\alpha).$$

The other terms in the expansions all contain y as a factor, and so in sum may be written as $y \cdot Q = (x - \alpha) \cdot Q$, where Q then is the quotient on division of $f(x)$ by $x - \alpha$.

i.e.
$$f(x) \equiv (x - \alpha) \cdot Q + f(\alpha);$$

and the remainder term $f(\alpha)$ has been found without either performing a long division or taking the step—now seen to be an irrelevance—of setting $x = \alpha$.

Jordanhill Training College, Glasgow.

A. G. SILLITTO

24. Chord of a conic with a given mid-point

It is known that the quadratic

$$k^2s_{22} + 2kls_{12} + l^2s_{11} = 0 \quad \dots (i)$$

gives the ratios $k : l$ in which the line P_1P_2 is divided by its intersections with the conic $s = 0$.

Take P_2 to be a meet with s of the chord having mid-point P_1 , and let A be the other extremity. Then $s_{22} = 0$, so that one root of (i) is $l = 0$. The other root is $k/l = -\frac{1}{2}$, since A divides P_1P_2 externally in the ratio 1 : 2. Hence $-s_{12} + s_{11} = 0$, which shows that P_2 lies on $s_1 = s_{11}$; this is linear and is satisfied by P_1 , and therefore is the required chord.

Technical College, Kingston-upon-Thames.

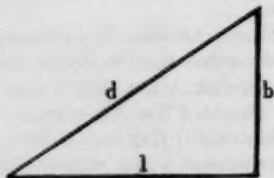
F. GERRISH

MATHEMATICAL NOTES

2766. Pythagorean Triads in Plimpton 322. (*Math. Gaz.* Vol. XXXIX No. 329 Sept. 1955, and Vol. XLI No. 335 Feb. 1957)

E. M. Bruins, after careful examination of Plimpton 322 in New York, has assured us, (*Math. Gaz.* No. 335, p. 26), that, "the first unit appearing on the later photographs, is the 'head' of the horizontal line between the rows, and that it is not a vertical wedge." He therefore interprets column I of the tablet as giving the values of b^2 , or $\frac{1}{2}[\lambda - (1/\lambda)]^2$, where λ has the 15 values listed in his last table, (p. 28), and where,

$$\begin{aligned} l &= 1 \\ b &= \frac{1}{2} \left(\lambda - \frac{1}{\lambda} \right) \\ d &= \frac{1}{2} \left(\lambda + \frac{1}{\lambda} \right) \end{aligned}$$



Had the first 'unit' been in fact a vertical wedge, then he would have to interpret column I as d^2 , or $\frac{1}{2}[\lambda + (1/\lambda)]^2$, since $1 + b^2 = d^2$. Neugebauer & Sachs earlier, (*Math. Cuneiform Texts*, p. 38), regarded the first unit as a vertical wedge, and thus interpreted column I as giving values of d^2/l^2 . If the first unit is not a vertical wedge, then Neugebauer & Sachs would regard column I as giving the values of b^2/l^2 . For the 15 triads of the tablet, they put,

$$\begin{aligned} l &= 2pq \\ b &= p^2 - q^2 \\ d &= p^2 + q^2, \end{aligned}$$

where p and q have the values given in their table, (*M.C.T.*, p. 40).

Now it is clear that by putting $2pq = 1$, both these sets of formulae become identical, providing $p/q = \lambda$. This is in fact the relation which does hold for each of the 15 sets of values in both *M.C.T.*, and Bruins' tables, so that the question whether the leading 'unit' is indeed a single vertical cuneiform wedge or not, becomes a different academic problem, not immediately pertinent, while both authorities are, in essentials, agreed on the basic theory behind the formation of these Pythagorean Triads.

There remains the problem of the mechanism of the errors which occur in Plimpton 322, the acceptance of any explanation, depending on its plausibility, having regard to the methods of the Babylonian scribe. These errors are,

1. Column III line 2 3, 12, 1 instead of 1, 20, 25.

- | | | |
|----------------------|-------------------|--------------------------------------|
| 2. Column I line 8 | 41, 33, 59, 3, 45 | instead of 41, 33, 45, 14,
3, 45. |
| 3. Column II line 9 | 9, 1 | instead of 8, 1. |
| 4. Column II line 13 | 7, 12, 1 | instead of 2, 41. |
| 5. Column II line 15 | 56 | instead of 28. |

Errors Nos. 2 and 3 we can dismiss as fairly obvious, No. 2 because the scribe wrote 59 instead of 45 and 14 separately, and No. 3 because the scribe inadvertently inserted an extra vertical wedge, putting 9 for 8. Error No. 5 is clearly the simple omission of a division by 2, either from the *M.C.T.*, values of $p = 9$ and $q = 5$, which give the triad, $1, 30 \mid 56 \mid 1, 46$, which on division by 2 gives the primitive triad, $45 \mid 28 \mid 53$; or from Bruins' values of $\lambda = 1; 48$, $1/\lambda = ; 33, 20$, giving the triad, $1 \mid ; 37, 20 \mid 1; 10, 40$, which on multiplication by 45, (or division by ;1, 20), gives again the primitive triad, $45 \mid 28 \mid 53$. Error No. 4, explained by the scribe writing n^2 for n , is both simple and plausible, whether the scribe was reading from a table of squares or using a formula. Bruins has to explain this error by suggesting that the scribe wrote, 2;24,20 in error for, 2;24,30, and then, wishing to multiply by 2, committed the further error of multiplying by 3 instead, then caps the lot with the final error of miswriting the product 7;13,0 as 7;12,1. This surely strains credulity beyond what may be regarded as reasonable.

Finally, error No. 1 is explained by Bruins, as the result of the scribe's intention of dividing two numbers by their common factors, 5, and that he had divided by 5 three times, when he should have stopped.

However the scribe is presumed to have kept going, on one of the numbers for two more divisions by 5, and then instead of getting the correct answer, 3,13,0, he committed the further error of writing it as, 3,12,1. Certainly this latter may be explained, as P. Huber of Zurich has suggested, in a letter to me in May 1955, as being due to an error of spacing, thus, $\overline{\text{III}} \text{ } \overline{\text{III}} \text{ } \overline{\text{III}} \text{ } \overline{\text{III}} \text{ } \overline{\text{III}}$. Nevertheless Bruins' explanation of this most important error, falls so clearly into the same category as his explanation of error No. 4, that I am quite unconvinced that it is superior to my own, given in *Math. Gaz.* Sept. 1955, Vol. XXXIX, No. 329.

R. J. GILLINGS

2767. Intersections of two parabolas

In Note 2637 (*Gazette*, XL, No. 334, 1956, p. 275) O. D. Mitrinović gives interesting expressions for the coordinates, with respect to rectangular axes Ox, Oy , of the fourth intersection $M_4(x_4, y_4)$ of two parabolas passing through three given points $M_1(x_1, y_1)$,

$M_2(x_2, y_2)$, $M_3(x_3, y_3)$ and having their axes parallel respectively to Ox and Oy . Use is made of biquadratic equations and Viète's formulae.

These coordinates may be obtained directly, if a very simple remark is taken into account.

It is well known that, in the quadrangle formed by the four intersections of any conic and a circle, each pair of opposite sides or diagonals have symmetric directions as to the axes of the conic. Hence, if P is the fourth intersection of one of the considered parabolas and the circle $M_1M_2M_3$, M_1M_2 and M_3P have symmetric directions as to Ox and so have M_2M_3 and M_1P . But this also occurs for the intersections of the second parabola and the circle $M_1M_2M_3$; therefore $P \equiv M_4$ and M_4 lies on the circle $M_1M_2M_3$.

According to the foregoing, the required coordinates of M_4 are obtained by solving the linear system

$$y - y_3 = -\frac{y_1 - y_2}{x_1 - x_2}(x - x_3), \quad y - y_2 = -\frac{y_1 - y_3}{x_1 - x_3}(x - x_2);$$

so, the sums Σ being extended to M_1, M_2, M_3 , we find by subtraction

$$x_4 \Sigma x_1(y_2 - y_3) = 2 \Sigma x_1(y_2 - y_3) - \Sigma x_1 \Sigma x_1(y_2 - y_3) \\ + x_2 x_3 \Sigma(y_2 - y_3).$$

This coincides with the result given *loc. cit.*, as the last sum vanishes.

We derive from the remark made above that the results remain the same when the two parabolas are replaced by any two centric conics having their axes parallel to Ox, Oy or by a centric conic having its axes parallel to Ox, Oy and a parabola having its axis parallel to Ox or Oy .

It may also be remarked that, when, in the Gauss-plane, the circle $M_1M_2M_3$ is taken as unit circle, and the unit-point chosen so that the diameter of that circle passing through that point is parallel to the axis of one of the considered parabolas and if the affixes of M_1, M_2, M_3 are denoted by t_1, t_2, t_3 then the affix of M_4 has the very simple value $(t_1 t_2 t_3)^{-1}$.

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R. GOORMAGHTIGH

2768. Nomograms with dividers and squared paper

P. M. d'Ocagne mentioned the construction of nomograms for use with dividers in his classic account "Exposée synthétique de la Nomographie", but little use has been made of the idea. Very simple nomograms of this class can however be constructed with the help of squared paper, as the following examples show.

A. The equation $a \cos \phi + b \sin \phi = 1$(1)

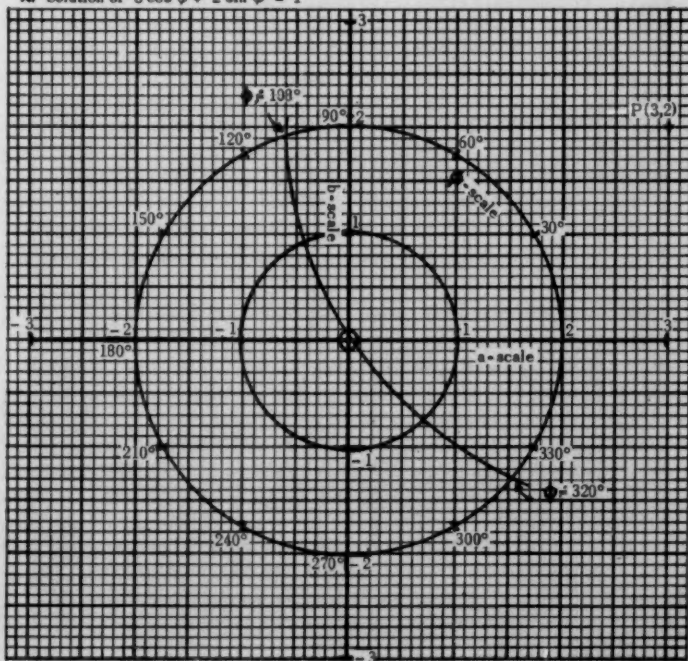
Put $x = 2e \cos \phi$, $y = 2e \sin \phi$, where e is a convenient unit of length. Then x and y satisfy the equations

$$x^2 + y^2 = 4e^2 \quad \dots(2)$$

and $x^2 + y^2 - 2aex - 2bey = 0 \quad \dots(3)$

(2) represents a fixed circle of radius $2e$ with centre at the origin O , while (3) represents a circle with centre at the point $P (ae, be)$ which passes through O . To construct the nomogram, graduate the axes with e as scale-unit and draw the circle with centre O and radius 2 units. Graduate this circle in degrees in the usual way: 0° at $(2, 0)$ and 90° at $(0, 2)$. Then with one point of the dividers fixed at $P(a, b)$ and with the distance between points PO , we can locate with the other point on the graduated circle the solutions of equation (1). Clearly if P lies inside the circle $x^2 + y^2 = e^2$ the equation (1) will have no solution; it will have just one if P lies on this

A. Solution of $3 \cos \phi + 2 \sin \phi = 1$



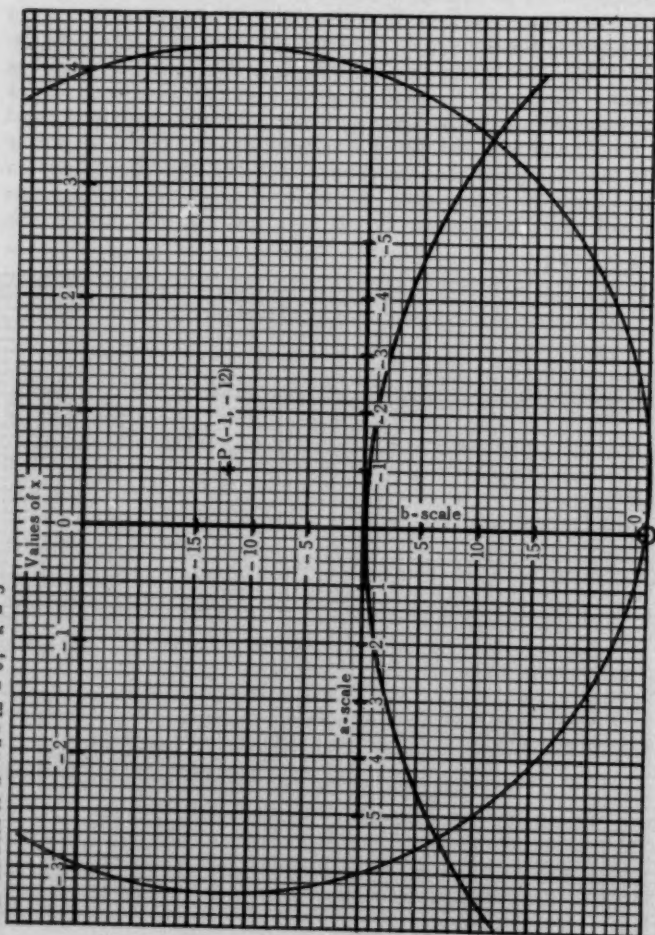
circle and two in all other cases. One advantage of this nomogram is that the quadrants in which the solutions of (1) lie are immediately obvious.

B. The equation $x^2 + ax + b = 0$(1)

This equation is satisfied by the abscissae of the common points of the straight line $y = k$ and the circle

$$x^2 + y^2 + ax + (b - k^2)y/k = 0 \quad \text{....(2)}$$

B. Solution of $x^2 - x - 12 = 0$; $k = 5$



which passes through the origin and has centre at the point $P(-\frac{1}{2}a, (k^2 - b)/2k)$. P is the point (a, b) referred to axes $x = 0$, $y = \frac{1}{2}k$ with suitable scales, which can be directly graduated with values of a and b . The solutions of (1) are complex if P lies inside the parabola $x^2 = k^2 - 2ky$, with focus at O and directrix $y = k$.

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R. LAUFFER

2769. A partial solution to a problem of D. J. Behrens

The problem* (motivated by an application to a field which I suppose should be called logistics) is to determine for what positive integers n the vertices of a regular $(2n + 1)$ -gon can be labelled with the marks $A_1, \dots, A_n, B_1, \dots, B_n, O$, in such a way that

- (i) no two segments $A_i B_i$ have the same length;
- (ii) no two segments $A_i B_i$ are parallel, and no such segment is parallel to the tangent at O to the circumcircle of the regular $(2n + 1)$ -gon.

Our object is to prove the following result, which gives a partial solution.

Such a labelling is certainly possible if $2n + 1$ is not divisible by any Fermat prime.

We first write the problem in a more arithmetic form. We label the vertices of the $(2n + 1)$ -gon consecutively, beginning at O , with the integers $0, 1, \dots, 2n$. Then if vertices A, B, C, D are labelled a, b, c, d it is easy to verify that the segments AB and CD are equal if $(a - b) \equiv \pm(c - d) \pmod{2n + 1}$, and parallel if $a + b \equiv c + d \pmod{2n + 1}$. The problem, then, is to determine whether the $2n$ non-zero residue classes mod $2n + 1$ can be divided into n pairs $(a_1, b_1), \dots, (a_n, b_n)$ such that

- (i) the residue classes $\pm(a_1 - b_1), \dots, \pm(a_n - b_n)$ are all different;
- (ii) the residue classes $a_1 + b_1, \dots, a_n + b_n$ are different from one another and from zero.

Suppose now that the distinct primes dividing $2n + 1$ are p_1, p_2, \dots, p_r , and that no one of them is a Fermat prime. Then, by definition, $p_i - 1$ is divisible by an odd number greater than 1, and there is therefore an integer k_i whose exponent mod p_i (i.e. the least integer m such that $k_i^m \equiv 1 \pmod{p_i}$) is odd and greater than 1. Evidently, $k_i \not\equiv \pm 1 \pmod{p_i}$. We choose an integer k such that $k \equiv k_i \pmod{p_i}$, $i = 1, \dots, r$. Then k has the properties that its exponent modulo any divisor of $2n + 1$ is odd and greater than 1, and that $k - 1, k$ and $k + 1$ are all prime to $2n + 1$.

Now let c be a non-zero residue class mod $2n + 1$. If m is the greatest integer such that the residue classes

* This Gazette, Vol. XLI, p. 101.

$$(i) \quad c, ck, ck^2, \dots, ck^{m-1}$$

are distinct, m is the exponent of k modulo some divisor of $2n + 1$, and so is odd and at least 3. Because m is odd, the residue classes

$$(ii) \quad -c, -ck, -ck^2, \dots, -ck^{m-1}$$

are distinct from those in (i). Thus we may begin our choice of pairs of non-zero residue classes by taking $(c, -ck)$, $(ck, -ck^2)$, \dots , $(ck^{m-1}, -c)$. This choice of pairs satisfies the following condition, (the second half because m is at least 3):

Every pair is of the form $(a, -ak)$; and if $(a, -ak)$ is a pair, $(-a, ak)$ is not.

If the residue classes in (i) and (ii) are not all the non-zero residue classes, we choose a non-zero residue class d not in either and repeat the process; and so on till the non-zero residue classes are exhausted. The pairs so chosen will continue to satisfy the condition just stated, and we show that they satisfy the required conditions also. First, the pairs (a_1, b_1) and (a_2, b_2) , where $b_i = -a_i k$, satisfy

$$a_1 - b_1 = \pm(a_2 - b_2)$$

only if

$$a_1(1 - k) = \pm a_2(1 - k),$$

which, since $k - 1$ is prime to $2n + 1$, implies that $a_1 = \pm a_2$. Since not both $(a, -ak)$ and $(-a, ak)$ are pairs, $a_1 = a_2$, and the pairs coincide. Using similarly the fact that $k + 1$ is prime to $2n + 1$, we see that $a_1 + b_1 = a_2 + b_2$ is impossible unless the pairs coincide, and that $a_1 + b_1 \neq 0$.

This completes the proof of the result. The proof provides, of course, a perfectly practicable method of constructing the pairs (a_i, b_i) in any given case. It is not to be expected that this sufficient condition for solubility should also be necessary, in view of the special nature of the pairs constructed. Indeed it cannot be, since Behrens has solutions for $n = 8, 10$, corresponding to $2n + 1 = 17, 21$, both divisible by Fermat primes.

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G. HIGMAN

Editorial note. A partial solution was also received from Dr. E. J. F. Primrose.

2770. Movements for Duplicate Bridge Competitions

Herr D. J. Behrens gibt einem kombinatorischen Problem eine geometrische Gestalt (Siehe *Math. Gaz.* May 1957, p. 101) Eine Arithmetisierung dieses Problems zeigt nun, daß eine gewisse Verwandtschaft mit dem schon lange bekannten Problem der n Damen des Schachspiels besteht.

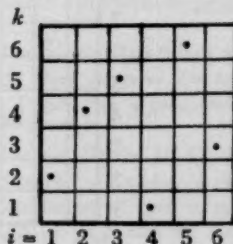
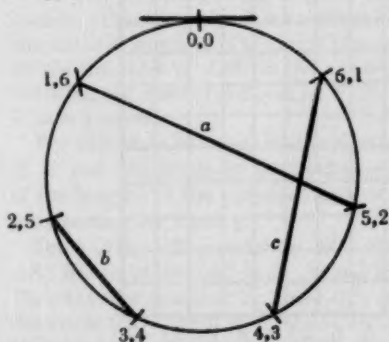
I. Die Arithmetisierung.

Werden die Punkte des regulär geteilten Kreises der geometrischen Veranschaulichung mit $(0,0)$, $(1,2n)$, $(2,2n-1) \dots (i, 2n-i+1) \dots (2n,1)$ in zyklischer Folge bezeichnet, dann gehören zu jeder Sehne zwei Zahlenpaare. Bei der Skizze ($n=3$) gehören zu den drei Sehnen die Paare (a) $(1,2)$, (b) $(2,4)$, (c) $(3,5)$, (c) $(4,1)$, (a) $(5,6)$, (c) $(6,3)$. Die am angeführten Ort angegebenen Bedingungen lauten jetzt:

Man bilde aus den Zahlen $1, 2, 3, \dots, 2n$

$2n$ verschiedene Zahlenpaare (i, k) , welche folgende Bedingungen erfüllen:

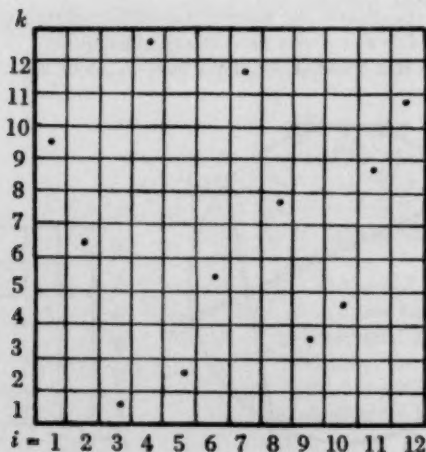
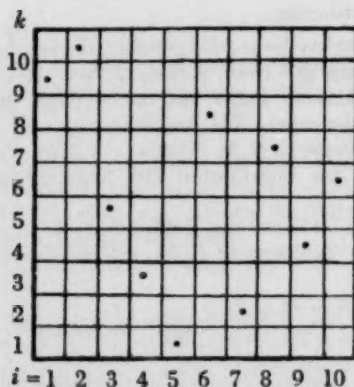
1. $i \neq k$, $i + k \neq 2n + 1$.
2. Gibt es das Paar (i, k) , dann gibt es nicht die Paare (i, j) und (j, k) .
3. Gibt es das Paar (i, k) , dann gibt es auch das Paar $(2n - k + 1, 2n - i + 1)$.
4. Gibt es das Paar (i, k) , dann gibt es nicht das Paar $(i + j, k + j)$ bei welchem $i + k + j$ von $2n + 1$ verschieden ist.
5. Gibt es das Paar (i, k) , dann gibt es nicht das Paar $(i + j, k - j)$.


 II. Die Veranschaulichung auf dem Schachbrett mit $(2n)^2$ Feldern.

Das Problem lautet jetzt:

Auf ein Schachbrett mit $(2n)^2$ Feldern sind $2n$ Steine zu stellen und folgende Bedingungen zu erfüllen.

1. Auf jeder Turmlinie steht genau ein Stein.
2. Die Hauptdiagonalen sind leer.
3. Die Stellung ist symmetrisch zur Hauptdiagonale $(1, 2n) \dots (2n, 1)$.
4. Ausgenommen die symmetrischen Steinpaare der 3. Bedingung steht auf einer Läuferdiagonale höchstens ein Stein.



Da das Problem der n Damen noch nicht gelöst ist, (sogar das Genie eines Gauss scheiterte daran) dürfte auch das obige Problem noch beträchtlichen Widerstand entgegensetzen.

Graz, Austria

R. LAUFFER

2771. On note 2664

Dr Maunsell's three problems are cases of the following: Given three parallel lines a, b, c in three-dimensional Euclidean space, to find a plane intersecting them in three points A, B, C forming a triangle similar to a given triangle A_0, B_0, C_0 . Take a plane π

normal to the lines a, b, c , and meeting them at A', B', C' . Locate C_1 in π so that the triangles $A_0B_0C_0, A'B'C_1$ are directly similar. The circle through C' and C_1 whose centre lies on the line $A'B'$ will cut this line in two points P' and Q' , labelled so that the order of the points on the circle is $P'C'C_1Q'$. Locate P'' on $C'P'$ so that the triangles $P''Q'C', P'Q'C_1$ are directly similar, the angles $P''C'Q', P'C_1Q'$ being corresponding right angles. Rotate the plane triangle $P''C'Q'$ about the line $C'Q'$ into the position $PC'Q'$ in which P' is the orthogonal projection of P on the plane π . Then the plane $PC'Q'$ will cut the lines a, b, c in a triangle similar to the triangle $A_0B_0C_0$.

This construction is well-known; see, for example, E. Müller, *Darstellende Geometrie*, 1. Auflage.

Graz, Austria

R. LAUFFER

2772. On note 2664

F. G. Maunsell has presented three equivalent problems for which he has not found an easy geometrical construction. I deal here with a generalization of his problem (ii).

To project (orthogonally) a given triangle into a triangle of given species. Denoting the former triangle by ABC , let the incircle of the latter triangle $A'B'C'$ touch the sides at D', E', F' , let D, E, F divide the sides of ABC in the same ratio, and let Σ be the ellipse touching the sides of ABC at D, E, F . Then it will suffice to project Σ into a circle.

For this we take the axis of projection l parallel to the minor axis of Σ and the angle of projection α such that $\cos \alpha =$ the ratio of the lengths of the principal axes of Σ . I now give a geometrical construction for l and α .

Draw AH, BK parallel to BC, AC to meet DE in H, K . Let AK, BH meet BC, AC in M, N , and let O be the mid-point of MN . Then O is the centre of Σ . Draw OP, OQ parallel to BC, AC to meet the circle OED again in P, Q , let DP, EQ meet in R and let XRY be a diameter of the circle OED . Then OX, OY are the principal axes of Σ .

Let OX, OY meet BC in U, V , draw UZ parallel to OD to meet OY in Z , and let either semicircle on VZ as diameter meet OX in G . Then (rejecting the trivial case in which $ABC, A'B'C'$ are already similar) two possibilities arise.

Either $OG < OU$, in which case we take l parallel to OY and $\cos \alpha = OG/OU$,

Or $OG > OU$, in which case we take l parallel to OX and $\cos \alpha = OU/OG$.

Oxford

J. G. MAULDON

2773. A transformation in eighth order magic square formulae

Recently I constructed a formula for eighth order magic squares by regarding each number from 1 to 64 as the sum of three numbers, one from each of the sets: 1, 2, 3, 4; 0, 4, 8, 12; 0, 16, 32, 48. A sample square so obtained is given in Fig. 1. Now suppose we attempt to derive this square from a formula in which each number is the sum of *two* numbers drawn from the two sets: 1, 2, 3, 4, 5, 6, 7, 8; 0, 8, 16, 24, 32, 40, 48, 56. The formula would then have to be that given in Fig. 2. We should expect certain relations to hold between the letters, and examination of rows, columns, diagonals and broken diagonals leads to the equations

$$A + H = B + G = C + F = D + E;$$

$$a + d = b + c = e + h = f + g.$$

Squares constructed from this formula, said to relate, do not necessarily have all the magic properties of the original formula, but the transformation is nevertheless an interesting one.

C. DUDLEY LANGFORD

FIG. 1

1	32	27	6	37	60	63	34
55	42	45	52	19	14	9	24
38	59	64	33	2	31	28	5
20	13	10	23	56	41	46	51
25	8	3	30	61	36	39	58
47	50	53	44	11	22	17	16
62	35	40	57	26	7	4	29
12	21	18	15	48	49	54	43

FIG. 2

<i>Aa</i>	<i>Dh</i>	<i>Dc</i>	<i>Af</i>	<i>Ee</i>	<i>Hd</i>	<i>Hg</i>	<i>Eb</i>
<i>Gg</i>	<i>Fb</i>	<i>Fe</i>	<i>Gd</i>	<i>Ce</i>	<i>Bf</i>	<i>Ba</i>	<i>Ch</i>
<i>Ef</i>	<i>Hc</i>	<i>Hh</i>	<i>Ea</i>	<i>Ab</i>	<i>Dg</i>	<i>Dd</i>	<i>Ae</i>
<i>Cd</i>	<i>Be</i>	<i>Bb</i>	<i>Cg</i>	<i>Gh</i>	<i>Fa</i>	<i>Ff</i>	<i>Gc</i>
<i>Da</i>	<i>Ah</i>	<i>Ac</i>	<i>Df</i>	<i>He</i>	<i>Ed</i>	<i>Eg</i>	<i>Hb</i>
<i>Fg</i>	<i>Gb</i>	<i>Ge</i>	<i>Fd</i>	<i>Bc</i>	<i>Cf</i>	<i>Ca</i>	<i>Bh</i>
<i>Hf</i>	<i>Ec</i>	<i>EH</i>	<i>Ha</i>	<i>Db</i>	<i>Ag</i>	<i>Ad</i>	<i>De</i>
<i>Bd</i>	<i>Ce</i>	<i>Cb</i>	<i>Bg</i>	<i>Fh</i>	<i>Ga</i>	<i>Gf</i>	<i>Fc</i>

2774. Halves and thirds

C is a point on a quadrant arc AB , centre O , and E is a point on the arc AC . The perpendicular from E to OB meets OC at F ; D is the foot of the perpendicular from C to OB and H is the foot of the perpendicular from E to OA . Then, if HF bisects OD at G , E trisects AC .

Let α, θ be the angles subtended at O by the arcs AC, AE . Then

$$\frac{OG}{OH} = \frac{EH}{OH - OF \cos \alpha} = \frac{\sin \theta \sin \alpha}{\sin (\alpha - \theta)}$$

and so, if

$$OG = \frac{1}{2} OD,$$

then

$$OG/OH = \sin \alpha/2 \cos \theta,$$

and therefore

$$\sin 2\theta = \sin (\alpha - \theta),$$

whence, since $\alpha + \theta < \pi$, $\alpha = 3\theta$.

ARCHIBALD J. FINLAY

2775. Sums of powers

Note 2687 and many other recent notes give interesting ways of showing $\Sigma r^3 = (\Sigma r)^2$. In this note a more direct and obvious method is presented. It is well-known that

$${}_1S_n = \sum_{r=1}^n r = \sum_{r=1}^n (n - r + 1).$$

$$\begin{aligned} \therefore {}_1S_n + {}_1S_{n-1} &= \sum_{r=1}^n (n - r + 1) + \sum_{r=1}^{n-1} r \\ &= \sum_{r=1}^n [(n - r + 1) + r] - n \\ &= n(n + 1) - n = n^2. \end{aligned} \tag{a}$$

Obvious of

$${}_1S_n - {}_1S_{n-1} = n \tag{b}$$

and

$${}_3S_n - {}_3S_{n-1} = n^3, \left({}_3S_n = \sum_{r=1}^n r^3 \right) \tag{c}$$

\therefore

$${}_1S_n^2 - {}_1S_{n-1}^2 = {}_3S_n - {}_3S_{n-1} \tag{d}$$

Putting $n = 1, 2, \dots, n$, adding and noting that ${}_1S_0 = 0 = {}_3S_0$, we get the required result

$${}_1S_n^2 = {}_3S_n.$$

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A. K. RAJAGOPAL

2776. A query

It is customary to define the Hermite and Laguerre polynomials for *positive values of n* . For convenience the differential equations and the usual Rodrigues formulae are given. (The notations are as in Erdélyi *et al.* "Higher Transcendental functions." Volume II McGraw-Hill 1953.)

Laguerre:
$$L_n(x) = e^x D^n(e^{-x} x^n).$$
$$xy'' + (1-x)y' + ny = 0$$

Associated Laguerre:
$$L_n^\alpha(x) = e^x x^{-\alpha} D^n(e^{-x} x^{\alpha+n}).$$
$$xy'' + (\alpha+1-x)y' + ny = 0.$$

Hermite:
$$H_n(x) = (-1)^n e^{x^2} D^n(e^{-x^2})$$
$$y'' - 2xy' + 2ny = 0.$$

D stands for (d/dx) and dash denotes differentiation.

It is a point of interest to put $n = -1$ in the above relations, to define $D^{-1} = \int_x^\infty \dots dx$, and to look for the consequences. Then the above formulae give

$$L_{-1}(x) = e^x \int_x^\infty \frac{e^{-x}}{x} dx = e^x E_1(x)$$

[Exponential Integral]

$$L_{-1}^\alpha(x) = e^x x^{-\alpha} \int_x^\infty e^{-x} x^{\alpha-1} dx = e^x x^{-\alpha} \Gamma(\alpha, x)$$

[Incomplete Gamma Function].

$$H_{-1}(x) = -e^{x^2} \int_x^\infty e^{-x^2} dx = -e^{x^2} \operatorname{Erfc}(x)$$

[Error Function].

By the well-known properties of $E_1(x)$, $\Gamma(n, x)$, and $\operatorname{Erfc}(x)$ we can show that the differential equations are indeed satisfied when we put $n = -1$ in those defined for positive n .

But if we define D^{-1} as $\int_0^x \dots dx$ we get slightly different results which do not satisfy the differential equations. The query then is; why does this particular way of defining D^{-1} alone fit in with the results correctly?

On this definition of D^{-1} we get the expressions for $H_{-n}(x)$ and $L_{-\frac{\alpha+n}{n}}^{(\alpha+n)}(x)$. For $H_{-1}(x)$ satisfies

$$y'' - 2xy' - 2y = 0$$

Differentiating this n times and putting y for $y^{(n)}$ we get

$$y'' - 2xy' - 2(n+1)y = 0$$

Comparing this with the equation for $H_n(x)$ we get

$$\begin{aligned} H_{-(n+1)}(x) &= \text{constant} \cdot D^n[H_{-1}(x)] \\ &= \text{constant} \cdot D^n[e^{x^2} \text{Erfc } x]. \end{aligned}$$

Similarly we can show that

$$\begin{aligned} L_{-(n+1)}^{(\alpha)}(x) &= \text{constant} \cdot D^n[L_{-1}^{(\alpha)}(x)] \\ &= \text{constant} \cdot D^n[e^x x^{-\alpha} \Gamma(\alpha, x)]. \end{aligned}$$

I wish to acknowledge my thanks to Prof. R. S. Krishnan for his encouragement.

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A. K. RAJAGOPAL

2777. Exponential function

On a growing amount $\pounds P$, a function of t years, the interest from time $t = 0$ to t at the rate of simple interest $\pounds r$ per \pounds per annum is

$$\int_0^t Pr \, dt. \quad \dots(I)$$

To find the total interest on $\pounds 1$ for t years at $\pounds r$ per pound per annum, the interest being added continuously. The simple interest on $\pounds 1$ for time $t = \int_0^t 1 \times r \, dt = rt$. This amount rt has been growing from 0 to t and the interest on it is

$$\int_0^t rt \times r \, dt = \frac{r^2 t^2}{2},$$

and the interest in $r^2 t^2/2$ growing from 0 to t is

$$\int_0^t \frac{r^2 t^2}{2} \times r \, dt = \frac{r^3 t^3}{3!}$$

and so on. Hence adding the original $\pounds 1$ the amount A at time t is

$$A = 1 + rt + \frac{r^2 t^2}{2!} + \frac{r^3 t^3}{3!} + \dots \quad \dots(II)$$

The relation to e^{rt} is found in the usual way from

$$dA = Ar \, dt$$

$$\log A = rt + C$$

so that if

$$A = 1 \text{ at } t = 0$$

$$\log A = rt$$

$$A = e^{rt}$$

Chester.

W. J. FEARNES

2778. Graphical discussion of the differential-equation

$$(D - \alpha)(D - \bar{\alpha})y = e^{i\nu x}.$$

Let p, q , be real numbers, $p^2 < 4q$. The differential equation $y'' + py' + qy = e^{i\nu x}$ can then be written in the form $(D - \alpha)(D - \bar{\alpha})y = e^{i\nu x}$, $\alpha = -\frac{p}{2} + i\frac{\sqrt{4q - p^2}}{2}$, $\bar{\alpha}$ denoting the conjugate of α .

The stable solution is

$$y = ze^{i\nu x},$$

$$z = \frac{1}{(i\nu - \alpha)(i\nu - \bar{\alpha})}.$$

Putting $z = ae^{i\varepsilon}$, (a, ε real), a is the amplitude and ε the phase difference of the forced vibration. It is

$$z = \frac{\alpha\bar{\alpha} - \nu^2 + i\nu(\alpha + \bar{\alpha})}{(\nu^2 + \alpha^2)(\nu^2 + \bar{\alpha}^2)},$$

therefore

$$\tan \varepsilon = \frac{\nu(\alpha + \bar{\alpha})}{\alpha\bar{\alpha} - \nu^2} = \frac{-p\nu}{q - \nu^2},$$

$$a = |z| = [(\nu^2 + \alpha^2)(\nu^2 + \bar{\alpha}^2)]^{-1/2}.$$

Here we have $|\nu^2 + \alpha^2| = |\nu^2 + \bar{\alpha}^2|$ and therefore $a = \frac{1}{|\nu^2 + \alpha^2|}$.

This relation can be understood in the complex plane. We plot

$$\alpha^2 = \frac{p^2}{2} - q - \frac{i}{2}p\sqrt{4q - p^2}$$

and through this point a line g parallel to the real axis, carrying a scale with ν^2 , starting from α^2 to the right. To any ν corresponds a point P on this line and the reciprocal of the distance between P and the origin O gives the corresponding amplitude. This reciprocal we get immediately when we draw the circle k , inverse to g with respect to the unit circle. The amplitude is then the length of the secant of this circle through PO . The further discussion is obvious.

University of Tasmania.

K. T. MOPPERT

2779. Coincidence

The following (insignificant?) oddity arose in setting a problem with numbers which were patterned, but otherwise random (the sign $^\circ$ denotes DEGREES):—

$$\sin^2 (6^\circ) = \cos^2 (12^\circ) \cong .3456,$$

$$\cos^2 (6^\circ) = \sin^2 (12^\circ) \cong .6543,$$

the approximation being very close.

E. A. MAXWELL

2780. On Pell's equation

If $C_n(x)$, $S_n(x)$ are both polynomials which satisfy the difference equation

$$T_{n+2} = 2xT_{n+1} - T_n$$

with $C_0 = 1$, $C_1 = x$, $S_0 = 0$, $S_1 = 1$

then $C_n^2(x) + (1 - x^2)S_n^2(x) = 1$

which we may verify by taking $x = \cos \theta$, giving $C_n(x) = \cos n\theta$ and $S_n(x) = \sin n\theta / \sin \theta$, or $x = \operatorname{ch} \theta$ giving $C_n(x) = \operatorname{ch} n\theta$, $S_n(x) = \operatorname{sh} n\theta / \operatorname{sh} \theta$.

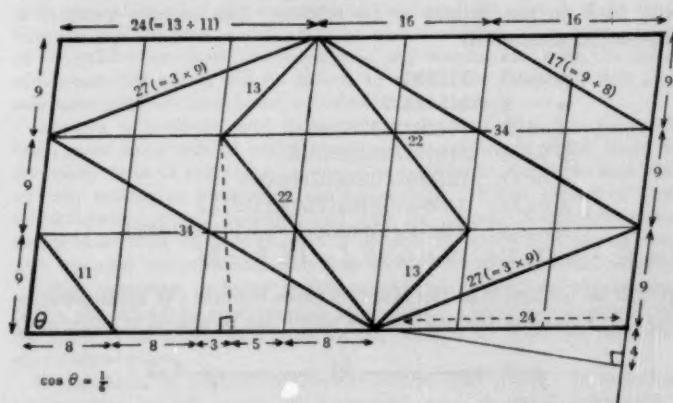
Hence if $X = x$, $Y = y$ is one solution in integers of Pell's equation

$$X^2 - kY^2 = 1$$

then $X = C_n(x)$, $Y = yS_n(x)$ is another, for any n .

Ingatestone.

B. E. LAWRENCE

2781. Parallelograms with integral sides and diagonals


C. DUDLEY LANGFORD

2782. On note 2722

The formulae of note 2722 are given in a paper by D. H. Lehmer in the *Canadian Journal of Mathematics*, 2, 1950.

H. G. APSIMON

2783. The representation of projectivities

E. J. F. Primrose, in Note 2681, *Gazette*, May 1957, p. 117, has considered the representation of the family of projectivities of a 1-dimensional projective space by the points of a 3-dimensional space. This idea has been developed by C. Stephanos, *Mathematische Annalen* 22 (1883), p. 299. See also *Encyclopédie des Sc. Math.* III 8, Géométrie projective, p. 130 and P. Libois, *Mathesis*, XLIV (1930), p. 121.

Fac. Polyt., Mons, Belgium.

R. DEAUX

PROBLEM

Years ago, my son, then a little boy, was playing with some coloured blocks. There were two of each colour, and one day I noticed that he had placed them in a single pile so that between the red pair there was one block, two between the blue pair, and three between the yellow. I then found that by a complete rearrangement I could add a green pair with four between them.

Clearly this is a perfectly general problem. For convenience we may denote the blocks by a pair of 1's, a pair of 2's etc. By experimenting with pieces of card cut as shown in the diagram, I have obtained the following solutions for n pairs; the other cases with $n \leq 15$ I do not believe to be soluble. Can anyone produce a theoretical treatment?

$$n = 3: 312132$$

$$n = 4: 41312432$$

$$n = 7: 17126425374635$$

$$n = 8: 3181375264285746$$

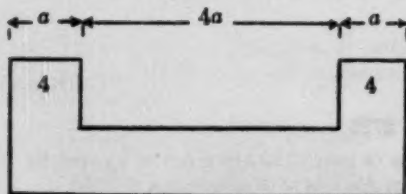
$$n = 11: 121e257t8395637e48t694$$

$$n = 12: Tt864e975468tT579e|312132$$

$$n = 15: F\theta e975fTt86579e\theta F68tTf|41312432$$

$$t = 10, e = 11, T = 12, \theta = 13, f = 14, F = 15.$$

It will be noticed that the last two cases contain the arrangements for $n = 3$ and 4 as separate groups, which can be placed at either end.



C. DUDLEY LANGFORD

REVIEWS

Elementary Mathematics. Part Four. By C. G. NOBBS. Pp. 384. 12s. 6d. 1957. (Oxford University Press).

In this book, which is the last of the series, the subjects of algebra, plane geometry and trigonometry, solid geometry, elementary co-ordinate geometry and calculus are developed with efficiency and thoroughness, almost from first principles. Only in the processes of arithmetic is reliance placed to any considerable extent on previous books of the series, continued in the present book as what may be termed applied arithmetic. The material, treatment and scope are ample for all parts of the "Alternative Syllabus" work, over the last eighteen months of the course.

By the inclusion of a wide variety of arithmetical topics from everyday life, by such algebraic and geometrical work as reveals a high degree of craftsmanship, by down-to-earth introductions to coordinate geometry and calculus, and by the rigorous exclusion of any kind of padding, the author has produced a book which may well be a leading example of its kind.

In his preface, Mr. Nobbs sets down the principles which have determined the nature of the present volume. Whereas the previous books have concerned themselves mainly with developing skill in a graduated sequence of mathematical tools, and the usage of these tools in suitable problems, part four moves towards the relations with one another of the branches of mathematics, and in some way towards the old idea of mathematics as a discipline. Very few points of the preface are open to challenge of any weight, and even the most opinionated of critics will be bound to admit the integrity, skill and scholarship which have been embodied in the book.

Algebra is reviewed and developed in the first fifty five pages of text, more than half of which consists of examples in which there is no heavy bulk of repetitive types. The bookwork is concise and the subject matter is interesting mathematically. Plane geometry and trigonometry occupy one hundred and ten pages. There are more riders than even an able pupil will be able to cover, and the section also includes mensuration, sine and cosine formulae, general values.

Solid geometry includes rectilinear figures, plans and elevations, paths on hillsides, approximate great circle distances, duration of daylight. This, along with work on the triangle of velocities, occupies thirty three pages.

The section on applied arithmetic covers fifty pages. It includes information on all types of household and personal arithmetic. Income tax, family allowances, P.A.Y.E., calorific values of diets all find a place, along with the more usual types of exercise on investments and scientific calculations associated with work and energy. Here is to be found much factual information, and the opportunity of bringing in the longer type of analytical example is frequently taken.

The necessary work on coordinate geometry having been covered incidentally in the section on algebra, the final fifty eight pages of

text are concerned with calculus. Here the craftsmanship of the author shows very clearly. The idea of the gradient of a parabolic graph is developed from tangency. Using h as the increment, the gradient functions of x^2 , cx^2 , and $a + bx + cx^2$ are found. There is an admirable balance between bookwork, routine types and longer examples. Maxima and minima, rectilinear motion theory and rates of change follow quickly. Integration is introduced as the problem of finding functions with a given gradient function, and is first used for the motion equations. Only when the integration rule is presumably working freely is integration used for area and volume. The standard symbolism is postponed until such time as its form cannot cause any trouble.

The book ends with a section of 105 miscellaneous examples covering the whole book, 22 pages of tables, answers to the exercises and a good index.

The setting up of the book and the profuse supply of well drawn diagrams are very good. With such a wealth of material in the book, the type and particularly the geometrical diagrams are rather smaller than in most textbooks. The section number is printed at the head of the page, with the title of the subject matter, and the page number is at the foot. This has undoubted advantages, but the index pages might well have carried the information that the numbers refer to pages.

Many teachers will wish to adopt this book for class use as opportunity allows. Until such time, it is well worth the price as a personal source book of ideas, examples and teaching hints.

J. W. HESSELGREAVES

Pure Mathematics. A First Course. By J. K. BACKHOUSE and S. P. T. HOULDSWORTH. Pp. 472. 12s. 6d. 1957. (Longmans Green).

Whilst the scope of this book is that of a generous first year sixth form book on analysis, the grading of the material also makes it very suitable for fifth form work. The work of the alternative advanced papers at ordinary level is covered in a very straightforward fashion. After the first chapter, which deals with coordinates and the gradient-intercept form of the line equation, calculus is taken up to the stage of finding centres of gravity of solids of revolution. The authors have included a number of their own ideas on presentation and sequence, in a sound and convincing fashion, whilst all the usual working techniques are suitably brought in.

A section on algebra includes surds, indices, logarithmic theory indicial equations, quadratic theory, remainder theorem, permutations, combinations, A.P. and G.P., series of powers, sigma notation, sums to infinity and the Binomial theorem. The choice of examples here is wise, for questions of technical difficulty have been avoided; at the same time there are a number of hints on further topics such as figurate numbers and their reciprocals, proof by induction, trinomial expansions.

The assumedly new work in trigonometry comes first. A start is made with the general angle, addition formulae, factor formulae. Solution of triangles and easy triangle formulae follow, and there is an exercise in three dimensional work. Radian measure, angular velocity and small angles then form an introduction to the derivatives of the ratios.

The deferred parts of coordinate geometry now appear, including conditions of tangency, loci, circle equations, common chords, angle and ratio formulae, polar coordinates. At the end come the perpendicular formula, parametric work and a short section on the pure geometry of the parabola.

There is a sense of urgency about this book which is attractive. A wide field of knowledge has been covered concisely yet fully. Most of the 19 chapters have a set of miscellaneous examples, many of which approach advanced level standards. In the preface, the authors have included brief suggestions as to how the book should be used. The printing is good, but there are a number of instances where an important heading is placed very near the bottom of a page.

J. W. H.

Calculus and Coordinate Geometry. By R. L. BOLT. Pp. 195. 10s. 6d. 1957. (Dent, London)

This book is designed for the abler pupils in fifth forms and for average pupils in first year sixth forms. Emphasis is placed on developing ideas by the working of numerical examples from first principles. New notations are frequently postponed when not immediately essential to the progress.

After introductory work on lines and gradients, calculus is developed up to the stage of volumes of revolution. The author has purposely placed in the second half of the book all the items which might impede smooth development of the calculus. This contains chapters on loci, the circle, section and perpendicular formulae, rates of change, products, quotients, trigonometrical functions.

The text and worked examples are well chosen and displayed. Each piece of new theory is followed closely by well graded examples. The layout is spacious and attractive. Twenty four-revision papers and ten test papers are provided, with many questions from ordinary level papers.

The pace of this book is very leisurely. Considering the number of pages, the restriction of the field in order to achieve greater simplicity hardly seems to be justified. The reviewer can find no mention of the usual formulae for the area of a triangle or the angle between two lines. Angle bisectors are found, but there is no hint as to which is which. Parametric coordinates have a paragraph, without any indication of their use. Somewhat greater nicety of approach and more indications of diversity of usage, would be an improvement.

There are three pages of formulae at the end of the book, and answers are provided.

J. W. H.

A New Certificate Arithmetic. By J. HISLOP. Vol. I. 7s. 6d. (with answers 9s.). 1957. (Methuen)

This is the first of 2 volumes designed to cover a four year course in Arithmetic at a Secondary School of any type in England or Scotland. The topics covered in Volume I include elementary work on the four processes, fractions and decimals, ratio, percentages, areas, volumes, simple interest, averages and the calculation of accounts. The book therefore assumes only the barest knowledge of Arithmetic and takes the reader up to the end of the second year of a Secondary School course.

What distinguishes this book from the large number of textbooks on the market at present? It has a large number of worked examples, well-graded; each chapter has a good selection of exercises suitable for pupils of varying calibre. The explanations are in general clear and concise, and the lay-out of the book is attractive. An introductory chapter gives an interesting account of the development of number notation, intelligible to a pupil of 12 or 13; a bibliography mentions four modern works on the History of Mathematics, at least one of which should be opened at some time even by pupils of this age. An interest in the history of mathematics cannot be awakened too early.

Tests of divisibility are stated—that for eleven is regrettably absent, as it is one of the most interesting and useful (and of course prompts the question—Why?) Quick methods of dealing with 25 and 125 are useful, although it is doubtful if the device often included in text books for decimalisation of money at sight is of any great practical value—pupils certainly rarely remember the rule. An excellent concise explanation of the operation of the Post Office Savings Bank will be of service to many adults as well as their children; it is one example of the modern treatment in this book—we no longer find that tea costs $2/6$ a lb. or coal $1/10$ a cwt., but that the "Week's Good Cause" brings in an average of £350, or that Zatopek ran 25 Kilometres in 1 hour, 19 mins., 11.8 secs. in 1952.

The chapter on Square Roots is well done. Square root tables often cause much difficulty and oral explanation is best. As far as possible, the written explanation of this chapter is intelligible; moreover the tables themselves are placed in the chapter and not relegated to the end of the book—an obviously convenient arrangement.

Some points of criticism may be made. In examples on fractions with several operations involved the order should be indicated by brackets—a rule to be remembered only imposes unnecessary work on the pupil. In the discussion of the area of a border, a piece of algebra leads to a general formula—the algebra is probably too difficult for the pupil and the formula is stated to be not essential. An attempt is made to explain the usual method for finding square roots—a written explanation of such an involved process may not be intelligible to a 13 year old. The simple interest formula is not used, each type of example being treated as an exercise in proportion. From the point of view of obtaining correct answers the formula is desirable and also provides a useful exercise on algebraic manipulation.

A few errors have been noticed; e.g. Chapter 7, example 6, 5047 dg. is not 5 kg. etc. In Chapter 12, there is some confusion in the first two lines as 2, 3, 5, 18 are not in proportion.

The author has obviously taken much pains to make the book attractive and up-to-date. As the first part of a G.C.E. "O" level course, it does its job successfully and can be recommended.

E. H. COFSEY

General Mathematics. By J. B. CHANNON and A. MCLEISH SMITH. Vol. 1. Pp. XIII, 303. 7s. 9d. 1957. Vol II. Pp. xiii, 323. 8s. 1957. (Longmans)

The authors of two well-known books dealing with arithmetic and algebra separately have now turned their attention to the production of a series of four volumes which will cover the whole of the mathematics required for the ordinary level examinations of G.C.E. The first two of these have now appeared. These provide insufficient material on which to judge the whole course, and one would have preferred to see the complete course to make a fair appraisal. However, the authors in their preface give assurances that the recommendations of the teaching committee of the M.A. have been kept in mind, and this is indeed evident in the first two volumes, so it is to be expected that the complete work will be a valuable addition to modern mathematical text books for the main school.

The task of presenting a unified mathematics course is not an easy one, particularly in the early stages. It is only after elementary ideas have been established that interplay between the traditional branches of the subject becomes possible. The early chapters of volume I show little continuity. But variety at this stage is essential if the interest of young pupils is to be held, and the authors' judgment on the size of each chapter is sound. They are not afraid to drop a topic and then to return to it later on when the pupils are capable of assimilating further developments. It is in the development of ideas that the value of this course lies, and in Book II it is quite clear that mathematical ideas are being linked firmly together. The two books contain all that would be needed to carry a pupil half-way through his main school mathematics course, and lend themselves more readily for use with classes which would reach O level in four years than for those taking five. The material has been carefully chosen and graded, and the exercises are plentiful enough for normal classroom needs. A good deal of space is given to worked examples, and while these may provide a basis for class discussion and some help for the boy working on his own, their use is rather limited. The authors have tried to represent on the printed page the exact form the solutions should take in the pupils' exercise books. But the arithmetical examples in particular do not carry much conviction. In the geometry bookwork, too, abbreviations which the pupil might use in writing out the theorems are used (including some that might be frowned upon by purists) not

always with the clearest effect. The sequence is, however, well planned and thorough. Early introduction to ratios and similar triangles enables a start to be made on trigonometry in Volume II. The impression is given that the course moves at a good pace over the elementary ground, covering points thoroughly but without over-elaboration.

The present volumes show that a good beginning has been made to provide a course of main school mathematics consonant with modern ideas. The remainder of the work is awaited with interest.

J. W. COLLEY

Solid Geometry. By J. S. HAILS and E. J. HOPKINS. Pp. vi, 196. 13s. 6d. 1957. (Oxford University Press).

It is an unfortunate fact that, although we live in a world of three dimensions, the scope of Geometry, both Pure and Analytical, of Trigonometry, and of practical constructions, should in the school sixth form be almost exclusively confined to the plane. It is time, in days when a pocket stereoscope can be bought for half-a-crown, that the tyranny of blackboard and paper was broken. But it presents the writer on Solid Geometry with a dilemma. Either he writes a monumental treatise in four volumes on (i) Practical Geometrical Drawing (ii) Pure Geometry (iii) Analytical Geometry (iv) Trigonometry in Three Dimensions, or he must attempt to include in one slim reasonably-priced volume such parts of these four disciplines which a sixth-former might reasonably be expected to know. This book adopts the latter course, except that the first subject finds no place in it, no doubt because books devoted to this are available already.

The treatment is of necessity therefore eclectic and a little sketchy. Most teachers will feel that the proper place for a number of these topics is to treat them as arising out of their two-dimensional context; the orthocentric tetrahedron immediately after the orthocentre properties of the triangle; the spherical sine formula after the plane one, and so on. But in our present set-up these swept-up remnants must be taken together, united only by the common element of "3-D." This book makes a good job of it. Here is most, if not all, of what every sixth-former should know of three dimensions. After an informal introduction the formal geometry of points, lines and planes, parallels and perpendiculars is developed, with judicious use of the plane at infinity (duly explained via central projection) to simplify the treatment. Chapter II on Polyhedra is the most miscellaneous; Euler's Theorem, similarity, mensuration, and a rather slight treatment of the formal geometry of the tetrahedron being all found in its 36 pages. Orthogonal Projection follows, and a largely mensurational treatment of cylinder, cone and sphere. Valuable chapters on Sections of a Right Circular Cone, (using Dandelin's Spheres and a little analytical work), and on Spherical Geometry, including Trigonometry and Map Projections, bring the book to a close. One could wish that these last chapters

could be more fully developed, since they form the most interesting part of the book.

Inevitably the choice of topics is a subjective matter, but the book, apart from the last two chapters, seems to have been influenced too much by current examination practice. It seems strange, for example, on p. 31 to mention the ruled quadric surfaces without at least giving a diagram of them and a short description of their generation. Vector methods might well have been used to give alternative proofs of the centroid theorems on pp. 50-51, and the condition $AB^2 + CD^2 = AC^2 + BD^2$ for the perpendicularity of opposite edges of a tetrahedron could well have appeared in the text on p. 55 instead of being relegated to an unworked example. Surely the properties of the orthocentric tetrahedron, even to its twelve-point spheres, form a valuable and interesting revision and extension of the corresponding triangle properties? They find no mention here. Would it not be better to include the construction for the orthogonal projection of any triangle into one of given species in place of the rather inconclusive discussion of the changes in the magnitude of angles on pp. 86-88? Again, the full discussion of the escribed spheres of the tetrahedron is no more elaborate than that accorded to the skeletal tetrahedron on pp. 128 ff, but it is declared to be "beyond the scope of the present volume." Strict three-dimensional coordinate geometry is also rigorously excluded, whether Cartesian or spherical polar, which is perhaps a pity. Doubtless one must draw the line somewhere, but in an effort to be really elementary the authors seem to have made the "ceiling" rather low.

There are a few errors that should be corrected in future editions. The statement on p. 31 that one and only one line can be drawn cutting three skew lines through any point on one of them is not true without exception. The congruence theorems which occur as examples on p. 35 are very "stiff" without any hints, and the third one is definitely false unless the face angle is the *included* face angle. The definition of a regular polyhedron on p. 42 is inadequate; there are four *convex* non-regular polyhedra whose faces are equilateral triangles, and if the word *convex* is removed the number is infinite. The example given on p. 44 of a non-Euler polyhedron invites confusion between the connectivity of the solid and the connectivity of a face. The statement on the same page that there cannot be more than five regular polyhedra needs the qualification of convexity, even under the given definition. Incidentally, here is another opportunity to open up a vista in few words, and to give the full total of nine under an extended definition. The student who started with an obtuse-angled triangle in Example 7, p. 57 would certainly be baffled! There is a misprint of P_2 for Q_2 in the second line of p. 87. When the authors take so much laudable care over elements of area and volume, it is a pity to subsume quite so much under the single word "Likewise" in the centre of p. 117. The condition on p. 130 for escribed spheres of a skeletal tetrahedron is not symmetrical and the conclusion is false. "Imaginary" points make an unexplained appearance on p. 143, and the confusion in the mind of an untutored reader might be made worse by the substitution of

"actual" for "real" in the footnote on p. 144. Fig. 102 does not appear to illustrate anything in particular, and it is not itself one of the described projections (or is it?).

The examples are nearly all chosen from public examinations, up to Scholarship standard, and are well distributed. Some of them involve ideas outside the range of the book itself. More could with profit have been worked in the text, but then either the price would go up considerably, or we should lose the discursive style which makes the book very suitable for a reader working by himself. No doubt the authors have chosen the better way. One need hardly add that in the hands of the Oxford University Press the diagrams and layout are exceptionally attractive.

H. M. CUNDY

Analytical Conics. By BARRY SPAIN. Pp. ix, 143. 30s. 1957. (Pergamon Press)

The justification which the author makes for adding to the long list of books on analytical conics is that he aims to present the basic ideas and methods and to omit unnecessary detail. In this he is very successful: many books on the subject extend to 300 pages or more, yet this book covers the ground adequately in little more than 100 pages.

My main criticism of the book is that, although the definition of coordinates in section 1 implies that the geometry is *real*, many results are given which are true only in *complex* geometry. For example, it is stated in section 15 that if $ab - h^2 > 0$ the equation $ax^2 + 2hxy + by^2 = 0$ represents two conjugate complex lines. Yet there are no such things as complex lines in the geometry defined by the author: the equation represents a single point. If complex points and lines are required in geometry, it is important that they should be introduced properly, and not allowed to creep in through the back door.

Whereas a parabola is defined by means of the usual focus-directrix property, an ellipse or hyperbola is defined as the locus of a point P such that $PM^2 = kA'M \cdot MA$, where A and A' are fixed points and M is the foot of the perpendicular from P to AA' . Surely it would have been preferable to define the three types of conic in the same way.

A minor criticism concerns the last chapter, on Generalized Homogeneous Coordinates. The main purpose of general homogeneous coordinates—one might almost say the only purpose—is to establish results in projective geometry, yet projective geometry is never mentioned. A beginner reading this last chapter may be puzzled about its significance.

The book is well written, well set out, and contains many helpful diagrams. There are plenty of examples, with solutions at the end. A student wishing to master the principles of the subject quickly would find the book very useful.

E. J. F. PRIMROSE.

Plane Coordinate Geometry and Trigonometry. (Second edition). By N. BERG-SONNE. Pp. 131. 1957. (Witwatersrand University Press)

This is really two books in one. The first part is a course on elementary trigonometry, treated by means of vectors. The second part is a course on plane analytical geometry, as far as the general equation of a conic: here vector methods play only a minor part.

The whole book has been written with great care. Every new term is properly defined, and the proofs are rigorous and clear. The application of vector methods to trigonometry is particularly successful.

The author, writing in South Africa, states that the book is intended for first-year university students, but hopes that parts of it will be used in schools. In Britain, first-year university students will have covered much of the ground already. However, the book would form an excellent basis for a school course.

E. J. F.

Circles. By D. PEDOE. Pp. x, 78. 20s. 1957. (Pergamon Press)

This interesting little book deals with various topics in mathematics in which circles play an important part.

Chapter 1 is devoted to some of the standard properties of circles, such as inversion and coaxial systems. Chapter 2 describes the representation of circles by points in 3-dimensional space: this treatment is clearly based on an article by the author in the Gazette (XXI, 210), but goes further.

I found Chapter 3 the most exciting in the book. By discussing those complex plane transformations of the form $w = (Az + B)/(Cz + D)$ which map the interior of a given circle on to itself, the author obtains the Poincaré model of hyperbolic non-Euclidean geometry. (According to Coxeter, this model is due to Klein, and the Poincaré model is obtained by inverting the circle into a line.)

Finally, Chapter 4 deals with the proof that of all closed plane curves of given perimeter the circle encloses the greatest area.

The book is intended primarily for university students. However, it would also serve the purpose of introducing some of the topics of higher mathematics to boys and girls before they go to a university, and for this reason it would be excellent for a school library.

E. J. F.

Algebraic Geometry. By C. V. DURELL. Pp. 388. 18s. 6d. 1956. Key 6s. (Bell)

The author's aim is to give a treatment of abstract algebraic geometry which will be a suitable preliminary to a University course and can, if necessary, be followed without supervision from a teacher.

In the introductory chapter spatial and abstract geometry are contrasted. Then, in the succeeding chapters, without further preamble, the author defines a point as a set of three homogeneous coordinates and develops, on an algebraic basis, an elegant and fairly

comprehensive account of the principles of plane projective geometry: there is one chapter devoted to solid geometry, showing how the plane geometry generalizes, and two others devoted to plane cartesian geometry. A pleasing feature is that the geometry is not obscured and frequently two (or more) proofs of results are given, to compare algebraic manipulation and geometrical reasoning. Scrupulous attention is paid throughout to the idea of dual elements and configurations and to the distinction between loci and envelopes. A great many examples are given at the ends of the chapters, for exercise, and more are worked in the text: the value of several of the examples is enhanced by their being susceptible to a geometrical, rather than an algebraic, treatment.

For the mathematical specialist this is an invaluable introduction to abstract geometry. Whether or not the book can be understood by an otherwise unaided student must naturally depend upon the student, but it is difficult to conceive how the task could better be undertaken.

There is a key to the examples in a separate volume. The solutions are abbreviated but sufficiently intelligible to aid anyone who tackles the problems with serious intent.

A. J. KNIGHT

Figures for Fun. By J. A. H. HUNTER. Pp. 128. 10s. 6d. 1957. (Phoenix House, Ltd.)

No wise teacher underestimates the stimulus which can be applied to a pupil through the medium of a puzzle, a joke, a trick problem, provided that the mode of solution, the explanation, the trick, are not entirely *ad hoc* but can be related to general principles. Commander Hunter has propounded many arithmetical puzzles and has now collected 150 of them into this book. They are generally of familiar type, but the dressings are almost always fresh and frequently diverting. A most valuable feature is that answers are given, and, with one or two exceptions, each answer is assigned a type symbol referring to one of 15 fully worked typical solutions. This should enable solvers to appreciate the importance of general concepts in mathematics, and so to derive instruction as well as entertainment from this pleasing little volume.

T. A. A. BROADBENT

The Basic Concepts of Mathematics. By K. MENGER. Part I Algebra. Pp. 93. (The Bookstore, Illinois Institute of Technology).

This account of linear and quadratic equations, complex numbers and polynomials is written for the adult reader. Menger discusses the difference between number and numeral, polynomial expression and polynomial. The emphasis is on the logic of operations, not the manipulations of signs. Important comparisons are made with language and logic which help to build insight, both in the student and the young teacher.

In the antithesis of sign and concept there is one surprising omission. Menger makes no reference to the concept of function; the *expressions* which form the context of the greater part of the book are *function signs*. For lack of this concept Menger seeks to analyse such an equation as

$$(x + 1)^2 = x^2 + 2x + 1$$

in terms of the classes of expression-values, instead of functional equality. But modern symbolic logic has found it necessary to distinguish between the equality of functions $f(x) = g(x)$, and the equality of the values $f(0) = g(0), f(1) = g(1), \dots$

R. L. G.

The Fascination of Numbers. By W. J. REICHMANN. Pp. 176. 15s. 1957. (Methuen)

This is a book for amateur mathematicians by an enthusiastic amateur, if an actuary may be so described. The sixteen chapters range over divisibility tests, sums of digits, arithmetical progressions, recurring decimals, congruences, fallacies and magic squares. Explanations are clear and there are some interesting novelties. The chapter on primes, reporting on an original investigation of the author's into a connection between sums of progressions of odd integers and primality, by one of the coincidences which are not uncommon in mathematics, partially formulates a theorem proved by A. N. Nicholson in the current (May, 1957) *Gazette*. The general solution in integers of $x^2 + y^2 = z^2$ is given in the form $x = a + \sqrt{2ab}$, $y = b + \sqrt{2ab}$, $z = a + b + \sqrt{2ab}$ for values of a, b which makes $2ab$ a square (but the values $a = 2kl^2, b = km^2$ for which $2ab$ is a square, are not given). The interest of the chapter on "digital roots" (sums of digits reduced modulo 9) would be greatly increased by including an account of the digit transformation game (*Gazette*, XXV, 265, pp. 156-159 and XL, 331, pp. 20-21), and a reference to J. S. Batty's article (*Gazette* XXXVIII, 324, pp. 90-95) would have valuably extended the chapter on recurring decimals.

R. L. G.

L'Aspect Moderne des Mathématiques. Par Lucienne Félix. Pp. 163. 800Fr. 1957. (Blanchard, Paris).

We are told repeatedly, in lecture and article, of the importance of giving the mathematics teacher in the school an insight into what is called modern mathematics, both for his own pleasure and for the pleasure and profit of his pupils. Mademoiselle Félix has written this book about modern aspects with this aim in mind and has achieved her object with great success. The book is as rich in the author's own experience of the problems of school teaching at all levels as she is competent to describe new ideas.

Amongst the topics discussed are mathematical logic, theory of sets, concepts of modern algebra, p -adic numbers, metric spaces and the theory of distributions. The book concludes with an account of a mapping of lines into points under which a cyclic quadrilateral becomes a parallelogram, which is applied to prove Miguel and Clifford's Theorems with remarkable economy.

I very much hope that this valuable book will soon find an English translator.

R. L. GOODSTEIN

Teach Yourself Statistics. By RICHARD GOODMAN. Pp. 239. 7s. 6d. 1957. (English Universities Press.)

Teach Yourself Statistics is a welcome addition to the well-known series of Teach Yourself books. It concentrates on the basic concepts of frequency distributions, probabilities, correlation and regression analysis, estimation from samples and tests of significance. The examples used to illustrate the text and those given at the end of each chapter cover a wide range of interests and the book ends with suggestions for further reading. Most readers who must teach themselves statistics will find this book more difficult than the calculus volume which is in the same series and to which frequent reference is made. This is inevitable. The mathematics of statistics is not easy mathematics. Mr. Goodman chooses his methods and his notation carefully, but he expects his readers to tackle difficult proofs.

This book is mainly concerned with the mathematical derivation of formulae and their application to numerical examples; there is little or no attempt to interpret the results. For example, the extensive treatment of correlation and regression analysis makes no mention of nonsense correlations, although numerical examples involving economic time series are included. The variance is described as a more satisfactory measure of spread than the interquartile range, though there must be many practical problems in which the converse is true. Also very few of the numerical examples include a discussion of the distribution of the variables involved, although as is made clear in the mathematical proofs, small sample theory is based on the assumption that the population to be sampled is normal.

The reason for these omissions is made clear in the preface. The author explains that "whether one likes it or not, Statistics is a branch of applied mathematics." He has therefore written an applied mathematics textbook. Anyone who likes his statistics this way will find Mr. Goodman's book both useful and interesting, but it is only fair to point out that there are other ways of approaching this subject. For many readers who must teach themselves statistics, a more practical treatment of the subject would have many advantages. Unfortunately, a knowledge of mathematical proofs and the ability to substitute numerical values in certain formulae do not prevent anyone from misusing or abusing statistics.

FREDA CONWAY

What is Science? Edited by J. R. NEWMAN. Pp. 493. 21s. 1956. (Gollancz)

This is a collection of 12 essays on various branches of science, including mathematics, physics and astronomy. The introductory essay, by Bertrand Russell, discusses the dangers of atomic warfare, secrecy in scientific research, and the hope of peace through the United Nations. The essay on mathematics is by E. T. Whittaker and has brief sections on Euclidean and non-Euclidean geometry, numbers and mathematical logic, of which the first two are admirably lucid but the third is liable to serious misunderstanding. How can one say, for instance, that Russell's paradox is inexpressible in symbolic logic when it is a contradiction that *formalised set theories* must be protected from? And what is the proposition found by Gödel in 1931 which has a meaning but cannot be either proved or disproved by means of any system based on axioms? Gödel showed how to construct an undecidable proposition in any given system of arithmetic, but the proposition changes with the system. Bondi's essay on Astronomy and Cosmology makes many important general observations on the scientific method in an exciting analysis of the assumptions underlying cosmological theories.

The essay on physics, a masterpiece of exposition, is contributed by E. V. Condon, an American nuclear physicist whose life vividly illustrates the social and economic dangers to which a scientist may be exposed in the atomic age. Condon tells the story of a seminar at Princeton when a graduate student was reporting on measurements of atomic masses and the energy of nuclear reactions in the nineteen thirties. In the audience was Einstein, enchanted to hear more and more numerical confirmation of his mass energy relation $E = mc^2$, grinning like a small boy and saying over and over again "Ist dass wirklich so!"

The final essay is by J. Bronowski on calculating machines. Bronowski distinguishes three of the mind's capacities, reason, learning by experience, and the discovery of general laws. The first two of these can be imitated on machines but the third is still the mind's sole prerogative.

Brief biographical notes on the contributors precede each essay and add to the charm of the many attractive expositions in the volume.

R. L. G.

Mathematics: A living language. By A. R. BIELBY. *Researches and Studies No. 16.* 3s. 9d. (Institute of Education, Leeds)

The author argues persuasively for the teaching of the understanding of mathematical ideas in our schools and cites a number of headings under which there appears much room for improvement. These are patterning, accuracy, functionality, idealisation, to mention only some. He stresses, although not always explicitly, the importance of selection, children should be made to realise that any concept whatever involves the selection of relevant and the rejection of irrelevant attributes.

Accuracy implies selection of the degree of accuracy, equations, identities and functional relations differ to the extent that the selection of values for the symbols is restricted, idealisation is again the process of selecting the relevant and manageable aspects of a problem in order to turn it into a mathematical problem at all. So too in Geometry, the conscious selection of a part of a figure, or of limiting or special cases, for consideration, works in favour of greater insight.

What is not apparent in the author's exposition is the practical way in which the conceptual framework with all its inevitable symbolism is to be anchored to the existing body of experience and knowledge of the learner, unless it be by the use of patterns as suggested under the first heading. It is just possible that research by practising teachers into this part of the learning process, i.e. optimum patternings of certain concepts and individual differences therein, would go far towards lightening the teacher's task during the stage of learning with which the present author is principally concerned.

Z. DIENES

Oxford Mathematical Conference. Pp. 111. 1957. (Times Publishing Co.)

These abbreviated proceedings of the Conference for Schoolteachers and Industrialists held at Oxford April 8-18, 1957 omit the 25 lectures by the lecturing staff of the Conference and report only the talks by visiting speakers from Industry. There is a very useful introduction to the use of electronic computers by G. E. Felton, of Ferranti, who has calculated π to 10,000 decimal places, on the comparatively slow machine *Pegasus*, in only 33 hours machine time; these 10,000 digits are reproduced in this volume. Another noteworthy talk was by Sir John Cockcroft on the mathematical work at Harwell. Most of the speakers stressed the need for more computational work in Degree Courses in mathematics. A discussion on scholarship Examinations (for University Entrance) was inconclusive but obviously enjoyed by all participants.

R. L. G.

Tables of Integrals and other Mathematical Data. By H. B. DWIGHT. 3rd Ed. Pp. 288. 21s. 1957. (The Macmillan Co: New York and London)

This is an admirable collection of results, well organised and easy to use. Some sections are of course much fuller than others. Integrals cover a great variety of integrands involving square roots of quadratics and trigonometric functions and there are long lists of expansions in series, including asymptotic expansions. Tables include trigonometric functions by 10 second intervals, the probability integral and Bessel functions in various ranges. The section on numerical integration seems rather limited, and there is not much point in introducing a section on divisibility tests which is confined to the divisors 3, 9, 2^n .

R. L. G.

Introduction to the Theory of Numbers. By L. E. DICKSON. Pp. 183. \$1.65. 1957. (Dover, New York)

Dickson's well-known textbook, originally published in 1929 and now reprinted without change by Dover, follows the general pattern of "Introductions" by seeking to familiarize the reader with the fundamental ideas in some special branch of the theory of numbers. The emphasis in the present case is on the theory of quadratic forms, a subject to which the author has made many original contributions.

The arrangement of the book is, to some extent, arbitrary. The first three chapters contain the standard material on congruences, primitive roots, and quadratic residues. Chapters 4, 6, and 8 deal with selected diophantine equations. In Chapter 11, the first three minima associated with indefinite binary quadratic forms are determined. Chapter 10, which stands somewhat apart from the rest of the book, presents the proof of the Thue-Siegel approximation theorem and Thue's theorem on diophantine equations with only a finite number of solutions. (The former result has been superseded by the recent work of Roth, an account of which can be found in the second volume of Le Veque's *Topics in number theory*.) The remaining chapters (5, 7, and 9) contain the hard core of the book. Here the theory of both definite and indefinite binary quadratic forms is developed, and such topics as reduced forms, automorphs, the representation problem, Pell's equation, and genera are treated.

The proofs are cast throughout in the classical arithmetical mould: there is no appeal to algebra nor to analytical techniques. But though the arguments employed are elementary, the reviewer doubts if this Introduction will fire the imagination of the beginner for whom it is intended. This is largely due to the difficulty the reader is likely to experience when trying to keep in sight the dominant ideas amid the welter of details. If he wishes to have a bird's-eye view of the subject, he would do well to supplement Dickson's work with at least a cursory perusal of the incomparable *Vorlesungen über Zahlentheorie* of Dirichlet-Dedekind.

L. MIRSKY.

Carrés Magiques. By A. DELESALLE. Pp. 70. 800Fr. 1956. (Gautier-Villars, Paris)

In the 70 pages of the work the author limits himself mainly to an exposition of the new magic square constructions which he has personally devised.

In the first part of the book certain generalities are treated. The square whose construction is sought, A say, is considered to be the resultant square of two auxiliary squares R and M which may be more easily constructed than the one sought. This idea of two more easily constructed auxiliary squares is carried right through the work. This replaces the entry of two digits in the same cell, as in Eulerian squares. Thus the compact construction of the older method is more spread out, and while the new construction is perhaps clearer it involves rather more labour.

This last section has in it a general method of construction simplified in particular from its inconveniences, by 3 solutions of special type.

The second part of the work relates to constructions similar to those explained in the first part, but the squares dealt with instead of containing the first N^2 natural numbers are made to hold other numbers than the natural set.

Senior scholars in grammar schools who wish to test their "technical" French and at the same time learn something of the making of magic squares would, I think, like to have access to this work in their school libraries.

W. J. CRATER

Statistical Theory. By LANCELOT HOGBEN. Pp. 510. 45s. 1957. (George Allen and Unwin).

Many statistical techniques in current use embody a mathematical model, a calculus of probability and a theory of uncertain inference. Statisticians agree about the calculus of probability, but there is no general agreement on the other two aspects. Professor Hogben contends that this represents a crisis and he sets out to examine the way by which mathematics enters statistical theory, mainly in the context of the biological sciences and social studies, and to pass judgement on the several schools of statistical inference. His approach is elementary at the mathematical level but a general knowledge of statistical ideas is most desirable in order that the reader may be stimulated without being overwhelmed—this is not a textbook.

Hogben begins by asserting an objective outlook, which requires all statements to be verifiable, and which he describes as behaviourist. He thus rejects theories of probability which are concerned with degrees of rational belief, and limits the use of inverse probability to experiments carried out in two stages. He considers that probability is meaningful only as a frequency ratio in an infinite random sequence, but he strongly and repeatedly objects to the widespread convention of conferring stochastic properties on natural phenomena by claiming that statistical data represent a random sample from some infinite hypothetical population. He shows how the normal law, in a close historical connection with the method of least squares, was translated from the field of error distributions to an all-embracing description of natural variation by Quetelet, seen as the skeleton in the cupboard of contemporary statistics. The resulting applications of regression are examined, and considered valid in physics, often unjustifiable in biology, and limited to classification in social studies. He disagrees with Fisher's ideas on significance tests and prefers, like Neyman and Pearson, to operate a rule which keeps the probability of incorrect decisions below a specified level.

Hogben has a great gift for lucid illustration, although it sometimes degenerates into verbal gymnastics and brilliant irrelevancies. He has written a lively account of statistical thinking, but it is not free

from bias, as when the views of Professor E. S. Pearson relating specifically to the work of inadequately trained statisticians in wartime (p. 27) are cited in a general disparagement of operational research; misconceptions, as when he asserts that the principle of maximum likelihood (p. 200) cannot be justified without recourse to inverse probability; and superannuated judgements, since the failure of confidence intervals in the problem of comparing two means (p. 444) was rectified by Welch in 1947. Many of his criticisms will be familiar to statisticians and they may feel disappointed with this book, which explicitly avoids a full discussion of current controversies on fiducial probability, and which finds fault with Neyman's theory of confidence intervals on grounds which lie outside any alleged crisis in statistical theory, but are merely concerned with matters of detail i.e. intervals of fixed length. In fact, Hogben is writing chiefly—but not exclusively—for non-mathematical users of statistical methods, whom the author visualizes as a vast army equipped with rule-of-thumb handbooks expounding the Fisherian drill. They certainly stand in need of a book like this one, and mathematicians whose training in statistics has been severely theoretical will also find it most instructive.

R. L. PLACKETT.

Introduction to the Geometry of Complex Numbers. By R. DEAUX. Translated by H. EVES. Pp. 208. \$6.50. 1957. (Ungar Publishing Co., New York)

Many elementary textbooks during the past thirty years have shown the simplifications which vectors bring to three dimensional geometry but the corresponding simplifications which complex numbers bring to the geometry of two dimensions has been largely neglected. This book fulfills therefore a real need, and presents an attractive and very readable account. The first chapter, perhaps the best in the book, deals with transformations, and with the cross ratio $\frac{z_1 - z_3}{z_2 - z_3} / \frac{z_1 - z_4}{z_2 - z_4}$; it is proved that the cross ratio is real if and only if the four points z_1, z_2, z_3, z_4 lie in a circle (or a line). The second chapter considers the equations of the line and circle, the conics and certain universal curves. The equation of the line takes the form $z = a + bt$, where t is a real parameter, and the line joining z_1, z_2 is

$$\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0.$$

The general equation of the circle is $z\bar{z} + az + \bar{a}\bar{z} + b = 0$, with centre at $-\bar{a}$, and radius squared equal to $a\bar{a} - b$. The third chapter studies the bilinear transformation $azz' + bz + cz' + d = 0$, the existence of double points and the invariance of the family of circles through the double points. Each section is followed by a set of exercises

(several of which are based on notes in *Mathesis*). Apart from the preface to the American Edition (for which the translator is not perhaps responsible) the translation is entirely successful. The printing is excellent.

R. L. G.

Introduction to Logic. By P. SUPPES. Pp. 312. 40s. 1957. (Van Nostrand, Princeton. Macmillan, London)

This is a most attractive first course in modern logic. After an introductory chapter on two-valued truth tables there is a very careful account of a system of natural inference. The system uses free and bound variables, and so-called ambiguous names which carry variable subscripts. Substitution for free variables does not occur (although the right to make substitutions is proved as a derived rule later in the book, and a forward reference would have been helpful) so that no restriction on conditional proof (the proof of $P \rightarrow Q$ by deriving a formula Q from a hypothesis P) is needed, but a free variable in a hypothesis is flagged until the hypothesis is discharged by a conditional proof. The derivation of $S(t)$, for a term t , from a universal sentence $(x)S(x)$ is subject only to the restriction that the term t may not contain a variable which becomes bound by a quantifier in $S(x)$; the converse derivation of $(x)S(x)$ from $S(x)$ is subject to the restriction that x is not flagged and is not a subscript of an ambiguous name. The derivation of $S(n)$ from $(\exists x)S(x)$, where n is an ambiguous name, is subject only to the requirement that ' n ' has not been previously used in the proof and that n carries as a subscript any free variable in $(\exists x)S(x)$, but the converse passage from $S(n)$ to $(\exists x)S(x)$ is subject to the conditions: (1) x is not a subscript or a bound variable occurring in $S(n)$; (2) x is not flagged. The restriction on generalisation, that x is not a subscript, is introduced to prevent such an inference as $(\exists y)(x)(x < y)$ from $(\exists y)(x < y)$ by means of the steps $x < \alpha_x$ and $(x)(x < \alpha_x)$; since $(x)(x < \alpha_x)$ is itself in no way undesirable, it would seem to be preferable to block the passage from $(x)(x < \alpha_x)$ to $(\exists y)(x)(x < y)$ by prohibiting existential generalisation on an ambiguous name with a bound suffix. There may be hidden dangers in universally quantifying a suffix in an ambiguous name, but the passage from $x < \alpha_x$ to $(x)(x < \alpha_x)$ appears to conform to standard mathematical practice, regarding α_x as a function.

The derivation theory is very well illustrated by examples, and there are numerous well graded exercises to provide the reader with practice in simple techniques, but there is no discussion of the completeness or consistency of the theory and no mention of a possible axiomatic foundation of predicate logic.

Amongst the topics treated in the second half of the book are the theory of definitions, (applied both to relations and function symbols); the language of the theory of sets (Cantor's theory), operations on relations and functions, and two applications of the axiomatic method, to a theory of probability and to particle mechanics.

In the section on naive set theory an opportunity is lost to draw attention to the duality between set theory and propositional calculus; there is a reference it is true to Boolean algebra but only in the set theory context. On page 29, priority in the discovery of the deduction theorem is given to Tarski but Herbrand has as good if not the better claim. On page 10 the convention that ' \rightarrow ' dominates ' $\&$ ' and ' \vee ' does not seem clear and the example illustrating the usage is confused by a misprint of ' \vee ' for ' $\&$ '.

One of the features of the book is an analysis of informal proofs which shows how standard mathematical practice deviates from logical formalism; this analysis is made, not to criticize the mathematician (who is seldom, if ever strictly formal even when he would claim this) but to facilitate the comprehension of quasi-formal proofs. The limitations of the formal method are not, however, mentioned and we are expected to have confidence in an axiomatic theory of real numbers without a hint of the consistency problem.

R. L. GOODSTEIN.

Vektoralgebra. By M. HIEKE. Pp. iv, 154. Mathematisch-Naturwissenschaftliche Bibliothek, 4. 1956. (Teubner, Leipzig)

This useful if pedestrian little book confines itself strictly to vector algebra; a second volume will deal with vector analysis. Vectors are physical quantities or line-segments, and there are a good many elementary physical applications. Noteworthy features are the careful attention to localised vectors; the recognition, in a somewhat heavy-handed fashion, of oblique axes and the reciprocal frame; the section on applications to crystallography; the matrix representation of a transformation and of a vector, and the connection with tensors. Scalar and vector products are denoted by "round" and "square" brackets; it is a little unfortunate that the brackets used for the vector product are almost indistinguishable from the modulus sign.

T. A. A. B.

Mathematical Logic. By R. L. GOODSTEIN. Pp. viii, 104. 21s. 1957. (Leicester University Press)

This book represents a rather remarkable contribution to the expository literature of mathematical logic. It is intended for mathematicians with little or no previous knowledge of logic, yet it proceeds to the heart of the subject and describes, clearly and with sufficient detail to be convincing to such persons, some modern results. At the same time it sketches more briefly many other aspects of the subject, so that a reader obtains a surprisingly adequate conception of mathematical logic as a whole.

How the author accomplishes this may be explained best by comparing the result with a picture. In such a picture there is a central area which

is in sharp focus, so that even relatively minute details are discernible; this central area blends gradually into a vague background. Furthermore every picture has necessarily a point of view; and from this point of view important features of the landscape may not only be dimly seen, but even be obscured altogether. A picture is not like a map, but often gives information which a map does not. The present book paints a picture of mathematical logic; as such it is something of a work of art.

For the reader who knows something about mathematical logic the content of this book may be described as follows. The focal point of the picture is the theorem on the incompleteness of arithmetic due to Gödel. Along with this there is a proof of the theorem of Skolem that arithmetic is non-categorical and the Church theorem on the undecidability of the predicate calculus. Leading up to this central point there is a fairly complete treatment of the testing of formulas of propositional calculus by truth tables, of the foundations of the predicate calculus—including a proof of the deduction theorem and the Gödel completeness theorem—and of primitive recursive functions. Other topics depicted in connection with this are the meaning of natural number (with which the author skilfully combines an explanation of the usual logical notation), various axiomatic and non-classical forms of propositional algebra, rules of natural inference, bracket-free notation, conversion calculus, logic-free formalization of primitive recursion, and the Quine system of extended predicate calculus. Among the topics treated inadequately or not at all may be mentioned the algebraic aspects of logic and the nonprimitive types of recursion associated with Kleene and his school. The formalization of primitive recursion is treated at rather great length because of the author's special interest in the question; he presents here an outline of a new version of such a theory.

The author claims that his book is self-contained in the sense that the major results are proved in detail. Although this is true in principle, there are, certain reservations about it. For one thing—the more technical results which constitute the background of the author's picture are often stated without proof. Since the treatment is discursive, these auxiliary threads are interwoven with the main ones so that it would be difficult to disentangle what is actually proved from what is dogmatically asserted. The central area indeed blends into the background. However this question is academic in relation to a book intended as an introduction. In principle the author gives enough detail to carry conviction to a not-too-sophisticated reader, and, what is more important, to give him a good idea of the nature of a logical proof. A reader who wants a more solid grounding of the major results than is given here will naturally turn to a more systematic and thorough treatment than one would expect in a book of this kind.

The reviewer has noted oversights as follows:

(1) In the proof of the deduction theorem (pp. 20 and 31) it is stated, in effect, that if B_1, B_2, \dots, B_n constitute a derivation of B from a hypothesis A , then $A \supset B_1, A \supset B_2, \dots, A \supset B_n$ constitute a derivation

of $A \supset B$ without hypothesis. This is not true; the proof shows that in general it is necessary to interpolate additional steps in order to obtain such a derivation.

(2) In the discussion of Gentzen rules on p. 26 it is stated that the omission of the rule

$$\frac{\neg \neg A}{A}$$

leads to a formulation of the intuitionistic algebra; this is followed by a mysterious statement about deducibility statements with a void consequent. This is either an error or an essential obscurity; for, as the reviewer interprets the situation, the indicated amputation leads to the minimal algebra rather than the intuitionistic, and there is no way of obtaining a deducibility statement with a void consequent.

(3) The nature of the restrictions imposed on certain variables on p. 34 is obscure.

(4) On p. 45 it is not clear what is to be done about $\sigma + \sigma$, $\sigma + \tau$, $\tau + \sigma$, $\sigma \cdot \sigma$ etc.

(5) The notes and bibliography at the end of the volume are inadequate. These notes are mostly confined to giving very brief indications of the sources of the ideas described in the text. Even for that purpose they might be amplified; for example the source of the proof of the Gödel completeness theorem should be stated explicitly. Aside from this, there is a chance that some reader may become so interested in the subject that he desires to pursue some phase of it farther; more help for such a reader would be desirable.

These criticisms merely point to blemishes in an excellent work. The book is, on the whole, very clearly written. It should be read rapidly with attention to the main ideas rather than the details. A mathematician who so reads it will no doubt obtain an insight into the nature of mathematical logic which is as adequate as he has any right to expect from a work of such small compass.

H. B. CURRY

Differential equations applied in science and engineering. By H. WAYLAND. Pp. xiii, 353. 1957. (Van Nostrand)

This book is *about* differential equations, rather than *on* them. It could hardly be otherwise, since in 350 very generously spaced pages it deals with vectors, ordinary and partial differential equations, special functions, Fourier series, boundary value problems, and the transform calculus. Technique and proofs are subsidiary to ideas, for the purpose of the volume is to bridge the gap between the engineer's or physicist's problem, and the details of solution of the mathematician's abstract—and generally simplified and approximate—formulation of the problem, a gap which often presents serious difficulties to those whose job it is to apply their mathematics. In the book, there is a wide variety of methods and applications, but the choice of so broad a field may have been a mistake, since in passing so quickly from one

topic to another the reader may feel that he does not come to close grips with any one of them. True, many books on differential equations fail to let the reader see the wood for the trees; Dr. Wayland's association with Harry Bateman has no doubt helped him to avoid this error and to provide a corrective for it in this broad survey. The student should be prepared to treat the book as a guide, and to seek among the references for further detailed information on particular topics. These references do of course indicate the familiar standard texts; but more valuable is the list of problems selected from recent periodical literature. These are graded into three groups, in increasing order of difficulty, and the reader is told briefly both the physical background and the mathematical techniques used in the paper. This is a most valuable feature, and I hope that if Dr. Wayland's book goes into further editions he will not fail to extend this section and keep it up to date.

T. A. A. B.

Elements of Partial Differential Equations. By I. N. SNEDDON. Pp. 176. 56s. 6d. 1957. (McGraw-Hill)

There is a general tendency for textbooks on partial differential equations either to concentrate on mathematical rigour with little regard to practical applications or alternatively to develop certain special techniques appropriate to a restricted range of physical problems. In "Elements of Partial Differential Equations" the author has found an excellent compromise and his book will be welcomed by all students who look for a balanced account of the subject. Problems encountered in many fields of science arise at various stages throughout the book. The basic physical ideas behind the governing equations are briefly discussed and the origin of the boundary value problems explained. But while the practical aspects are given due prominence, it should be emphasised that the subject is approached essentially from the mathematician's point of view, and that applications to physics are used primarily as illustrations of the behaviour of given solutions and as examples to which the methods of solution can be applied. The book is written at and slightly beyond the level of an advanced honours course and it contains a comprehensive survey of the mathematical techniques available for solving boundary value problems.

The first two chapters deal with ordinary differential equations and first order partial differential equations. Geometrical principles are extensively used but only in so far as they lead to a clearer understanding of the fundamental ideas, some of the more specifically geometrical concepts being relegated to an appendix. Different methods of solution are carefully enumerated and, as in every other section throughout the book, fully illustrated by worked examples. The applications include a discussion of the Hamilton-Jacobi equation in Dynamics and of equations occurring in the theory of stochastic processes. There is also an elegant exposition of the elementary principles of thermodynamics depending upon Carathéodory's theorem.

Second order equations are reviewed in general terms in Chapter 3.

The author has a difficult task in attempting a survey of such scope and it is perhaps inevitable that alternative methods of presentation should suggest themselves. For example it might be shown more clearly how the basic Green's identity

$$vL[u] - uM[v] = \frac{\partial H}{\partial x} + \frac{\partial K}{\partial y}$$

is the source from which in turn Riemann's method for hyperbolic equations and the Green's function technique for elliptic equations take their origin. Further, the fundamental part played by characteristics in the solution of hyperbolic differential equations is given comparatively little emphasis either here or in the later chapter on the wave equation. Nevertheless the chapter contains a great deal of very important material and it includes, for example, sections on the operator method for equations with constant coefficients, on the method of integral transforms, on linear equations in three independent variables and on non-linear equations.

In the last three chapters on Laplace's equation, the wave equation and the diffusion equation respectively, clear and detailed accounts are given of the principal methods of solution. Emphasis is laid on the importance of the elementary solution in relation to problems in elliptic and parabolic equations. It is to be hoped that the use made of integral transform techniques will stimulate instruction in a method of solution which is in general insufficiently taught at an undergraduate level. It is of particular interest to note that in these chapters a number of modern techniques of a more advanced nature have been included. Thus, for example, there are sections on fractional integration, on certain dual integral equations occurring in potential theory and on contour integral methods of solving Riemann's equation of one-dimensional gas dynamics.

A very large number of examples for the student is included and these are designed both to develop his application of methods described in the text and to extend his knowledge of fundamental mathematical and physical principles. It is unfortunate that, both in these examples and in the text, numerous minor errors and misprints are to be found.

There is no doubt that the author has made a most important contribution to the literature on partial differential equations and has offered a book which deserves to be widely read by students of applied mathematics and by research workers in a large variety of subjects.

A. G. MACKIE

Ebene und Sphärische Trigonometric. By G. HESSENBERG and H. KNESEH. Vth. Ed. Pp. 172. DM 2.40. 1957.

Höhere Algebra. By H. HASSE. Vol. I, Lineare Gleichungen. IVth Ed. Pp. 152. DM 2.40. 1957. (Sammlung Götschen, 99, 931. de Gruyter, Berlin)

The attractively presented summary of plane and spherical trigonometry includes the trigonometry of the triangle and quadrilateral

in considerable detail, with numerous worked examples. Complex trigonometry is introduced in terms of two-dimensional vectors; the treatment of infinite series is heuristic. Hasse's algebra is too well known to need further recommendation. A translation from the Third Edition has been published in the Chelsea series, but of course this production is far less expensive.

R. L. G.

Mathematics. By A. T. STARR. Pp. ix, 547. 45s. 1957. (Pitman)

Dr. Starr sets out to cover, in one volume, precisely the syllabus of mathematics required for Parts 1 and 2 in the engineering degree of the University of London. Perhaps no one can be more aware of the limitations imposed on a book by such a programme than the author himself. Room must be found for algebra, complex numbers, infinite series, calculus from the very beginnings up to multiple integrals, differential equations, Fourier series, Laplace transforms, matrices, vector algebra and analysis, coordinate geometry, as well as for applied mathematics from elementary statics up to bending of beams and the beginnings of fluid dynamics; for good measure there must be a chapter on probability and statistics. All main topics must be fully illustrated by worked examples, and the space occupied by some five or six hundred exercises for the student is not negligible. It follows that the main text must be severely compressed, and this compression sometimes leads to inadequacy, as when uniform convergence is introduced and dismissed in 6 lines, sometimes to error, as when the Cauchy-Riemann equations are said to be a sufficient as well as a necessary condition for a function to be regular. But a more serious defect is the lack of motivation; there is no room to explain why a method is important or what its place in the main stream of development is. Thus, in spite of the care with which the worked examples have been chosen and the fullness of detail in the steps of the solutions, the student working alone would not make easy progress; as a class-book, in the hands of an experienced teacher, the book would have much more value.

It must be emphasised that this is criticism of the task the author has set himself, rather than of his mode of performing that task. Even so, he would have done it better with more elbow-room at his disposal, obtaining this perhaps by two 300-page volumes, one for Part 1 and one for Part 2.

T. A. A. BROADBENT

Advanced Calculus. By R. CREIGHTON BUCK. Pp. 423. \$8.50. 1956. (McGraw-Hill)

Published in the International series in Pure and Applied Mathematics, the book provides a systematic and modern account of advanced

calculus addressed to Mathematics, Physics or Engineering students. Real numbers and their properties are assumed, though an axiomatic treatment is sketched in an appendix. Convergence, integration and differentiation of functions of several real variables are studied and the associated techniques are carefully explained and illustrated by worked examples. The methods are applied to the study of curves and surfaces and to the determination of necessary conditions for extrema. The final chapter reviews vector calculus, introduces differential forms, discusses Stokes' theorem and ends with a section on the calculus of variations. There is an adequate supply of exercises with answers or hints for solution, and an index. With this modest scope, there is no great proliferation of facts and formulae. Instead, the book offers degree students a firm foundation in basic techniques.

The exposition is clarified by taking the case of two or three variables as typical. Arguments are illustrated by well-drawn diagrams. Notable features are the characterization of the integral of a continuous function as a uniformly differentiable additive set function, the use made of the fact that a differential is a linear transformation and the introduction of exterior differentiation of differential forms. The latter affords a unification of the theorems of Gauss, Green and Stokes with consequent gain of insight. Statements of theorems are usually reliable, though a number of the proofs are faulty. The author is not alone in being unreliable on the ill-fated subject of functional dependence and the proof of Stirling's formula (pp. 160-2) reads like a qualified analyst being careless. The continuity of the inverse of a transformation is assumed without proof (p. 217) and the continuity of the Jacobian needs to be used to deduce the change of variable rule from a lemma (p. 239). The first proof of the theorem on conditional extrema (p. 295) rests on a trivially false parenthetical remark whilst the second uses an unproved fact from the section on functional dependence. Lagrange's undetermined multiplier rule (pp. 296-7) is deduced by an argument of the form $A \subset C$ and $B \subset C$ hence $A \subset B$, and is wrongly stated to give *all* conditional extrema. The reader will be left in doubt how he is intended to complete the proof of a further theorem on the change of variable rule (p. 345), and some of the hints for solution of exercises are incorrect. There are also quite a number of misprints.

Despite these and other faults, the reviewer still feels that the book serves a useful purpose for students and teachers alike.

W. F. NEWS

Mathematical Analysis. By T. M. APOSTOL. Pp. 553. 1957. (Addison-Wesley. Reading, U.S.A.)

The subtitle of this book, a modern approach to advanced Calculus, is an apt description of the work. The contents range over integration theory, theory of series, vector analysis, Fourier series and transforms, and complex function theory. Many of the chapters are strikingly fresh and vigorous, the presentation bold and discussion

in considerable detail, with numerous worked examples. Complex trigonometry is introduced in terms of two-dimensional vectors; the treatment of infinite series is heuristic. Hasse's algebra is too well known to need further recommendation. A translation from the Third Edition has been published in the Chelsea series, but of course this production is far less expensive.

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illuminating. Starting with an axiomatic foundation for number theory which postulates the existence of the least upper bound of a bounded set, there follows a rapid survey of the foundation theorems of analysis. The Heine-Borel theorem is used to show that every closed and bounded set is compact, and many results on continuous functions which are classically proved for intervals are here established for compact sets. A generalised form of Taylor's Theorem for a pair of functions is proved. The account of the Riemann Stieltjes integral is unusually full and extensive. Complex variable function theory is introduced early in the text. Topological problems in connection with Cauchy's theorem are carefully explained and a good deal of detail is given (excluding a proof of the Jordan curve Theorem). In connection with Cauchy's theorem it is disappointing to find the result itself proved on the redundant hypothesis that the partial derivatives are continuous.

The chapter on infinite series contains an interesting proof of a theorem of S. Bernstein to the effect that if a function f has continuous derivatives of any order in an open interval (x, b) which contains the point a and if f and all its derivatives are non-negative in the interval $a \leq x < b$ then the Taylor expansion about every x_0 in $a \leq x < b$ is valid for $x_0 \leq x < b$.

The proof of Theorem 4-20 on page 73 would be made clearer by distinguishing the cases $a \in S$, $a \notin S$ at the outset and showing that the latter leads to a contradiction. The proof given of the classical sufficient conditions for a minimum of a function of several variables (p. 150) is invalid since the conditions $aA > 0$, $bB > 0$, $cC > 0$ do not ensure that $ah^2 + 2bhk + bk^2$ and $Ah^2 + 2Bhk + Ck^2$ have the same sign for all small enough $|h|$, $|k|$.

The chief blemish in the text lies in the theory of differentials where there appears to be a confusion between the condition for $\Sigma(\partial f/\partial x_1)dx_1$ to be a differential (i.e. a differentiability condition) and the *existence* of a differential, for $\Sigma(\partial f/\partial x_1)dx_1$ exists when the partial derivatives exist and *is* the differential when the function is differentiable. All that is needed is a change in emphasis in the formulation.

R. L. GOODSTEIN

Berechnung magnetischer Felder. By F. OLLENDORFF. Pp. x, 432. 1952. £5. 12s.

Elektronik des Einzelelektrons. By F. OLLENDORFF. Pp. xii, 643. 1955. £8. (Springer, Vienna)

These volumes, the first in Ollendorff's "Technische Elektrodynamik," cannot be recommended too highly.

The first gives a comprehensive collection of detailed solutions of the magnetic field problems arising in power engineering and in instrument design. Real and complex variable methods of solving scalar potential problems are discussed in chapters one and two; the remaining two

chapters deal with vector potential fields and electrodynamic forces.

The second volume gives an equally detailed treatment of a wide range of problems concerning the motion of the free electron, both from the standpoint of classical mechanics and from that of special relativity; the subjects of particle accelerators and electron optics are covered very fully.

Both books are outstanding for their clarity and are beautifully produced. They are expensive, and the mathematical detail could often be trimmed; nonetheless, they are essential reference books for workers in these important fields. Either could form the basis of a thorough postgraduate course.

J. W. REED

Initiation à la Logique. By LE R. P. DUBARLE. Pp. 89. 1400Fr. 1957. (Gauthier-Villars, Paris)

This is a first rate introduction to the concepts of symbolic logic. Sentence logic is set up both by truth tables and as an axiomatic system and the concepts of completeness and decision procedure are explained, including an arithmetical decision method (due to Herbrand). The character of predicate logic is carefully described (using metalogical symbols explicitly) and an axiom system is set up. (In axiom 2.22 the negation sign should be deleted.) This is followed by a brief section on class logic. Other topics briefly discussed are intuitionistic logic, modal logic and combinatorial logic; there is also a valuable account of Gentzen's sequence logic (unfortunately handicapped by some misplaced dots in the inference schemata for conjunction). There has been wide publicity given in recent books to Gentzen's first method of natural inference but I think Dubarle is quite right in choosing the sequence calculus for his exposition. Natural inference is hedged around with so many restrictions and limitations and suffers from such serious notational difficulties that the clear cut sequence logic is much to be preferred.

The final section on the great theorems of modern symbolic logic is too condensed to be entirely acceptable but it may perhaps convey something of the gist of the results. Gödel's Theorem on incompleteness cannot be appreciated without an account of the underlying recursive function theory (or something comparable) and Dubarle's text makes no reference to this. He appears to be following Gödel's original account (not the symmetric form of the proof devised later by Rosser) but the need for ω -completeness is overlooked. And in discussing the impossibility of a proof of freedom from contradiction the condition for freedom from contradiction, that for all, m, n, p it is false that m is the number of the proof of sentence p and n is the number of the proof of not- p , is apparently taken in the form "for some p and all m, n etc."

R. L. GOODSTEIN.

BRIEF MENTION

Survey Adjustments and Linear Squares. By H. F. RAINSFORD. Pp. 326. 50s. 1957. (Constable)

A theoretical section on the method of least squares precedes an account of applications of the method to triangulation and other survey problems.

Calcolo Delle Matrici. By S. CHERUBINO. Pp. 322. 1957. (Edizioni Cremonese. Rome)

The formal theory of matrices with applications to analysis.

The Principles of Quantum Mechanics. By P. A. M. DIRAC. 4th. Ed. Pp. 312. 35s. 1958. (Oxford University Press)

For this edition the chapter on quantum electrodynamics has been rewritten; a theory which demands conservation of the number of charged particles is out of touch with physical reality, says Professor Dirac in the Preface, and has been replaced by a quantum electrodynamics which includes creation and annihilation of electron-positron pairs.

Statistical Exercises. Compiled by N. L. JOHNSON. Part II. Analysis of variance and associated techniques. Pp. 107. 12s. 1957. (University College London)

A sequel to the more elementary first volume issued in 1953.

Worked Examples in Electrotechnology. By A. C. SHOTTON. Pp. 240. 12s. 6d. 1958. (Harrap)

The worked examples are followed by graded exercises. Topics considered include A.C. circuits, Electromagnetism, D.C. machines and Illumination.

Elementary Coordinate Geometry. By E. A. MAXWELL. 2nd Ed. Pp. 336. 25s. 1958. (Oxford University Press)

The main change in this new edition is the addition of a chapter on the general conic. The review of the first Edition by P. M. Hunt was in M.G. XXXVII, p. 300.

Geometrical Drawing. By I. H. MORRIS. Pp. 228. 9s. 6d. 1958. (Longmans, Green.)

The 14th Ed. of this standard reference book for Art Students has been edited by J. C. Scott.

BRANCH REPORTS

NORTH EASTERN BRANCH

REPORT FOR THE SESSION 1957-1958

Officers:

President, Dr. M. Hutton; *Secretary*, Mr. J. F. Reed; *Treasurer*, Miss. J. A. Scott.

There are about 60 members.

The following meetings were held during the year:

Members of the branch paid a visit to the Grammar-Technical School for Boys, South Shields, where the Head Master, Mr. W. E. Egner, gave an address on "Operational Research and the Grammar Schools."

A meeting was held at Sunderland Technical College, when Dr. M. Hutton, the President, spoke on "The Romance of Mathematics."

Dr. E. S. Page, Director of the Durham University Computing Laboratory, demonstrated the Ferranti Pegasus computer in the Laboratory at Newcastle and gave a talk on "Automatic Digital Computers."

Meetings were held at Durham in conjunction with the University School Examinations Board. In the morning the speaker was Mr. J. H. Smith, H.M.I., who spoke on "The Teaching of Mathematics in Scotland." The afternoon meeting took the form of a symposium on the teaching of mathematics.

F. REED, *Hon. Sec.*

QUEENSLAND BRANCH

REPORT FOR THE SESSION 1957-1958

The Association records with regret the death in April, 1957, of Mr. J. C. Deeny, B.A., F.R.G.S., who had been a member for more than thirty years. At the time of his death Mr. Deeny was an honorary member.

The Annual Meeting was held at the Dental College, Turbot Street, on 19th March, 1957. The Annual Report and the Statement of Receipts and Expenditure were presented to the meeting, and both were adopted.

This was followed by the election of officers for 1957, and then by the Presidential Address, entitled "Some Problems in Plane Trigonometry," and given by the President, Mr. E. W. Jones, B.A.

Three general meetings were held during the year, all at the

Dental College, Turbot Street. At the first, on 21st May, Associate-Professor J. P. McCarthy read a paper entitled "Galileo's Dynamics." At the second and third meetings, on 29th July and 23rd September, the topic of discussion was "The Teaching of Calculus at Secondary School Level." Talks were given by Messrs. Young, Hubbard, and Rev. Br. Miller, and the Association is gratified that on both occasions so many guests accepted invitations to be present.

The Statement of Receipts and Expenditure for the year discloses a credit balance of £8 13s. 8d. Most of the expenses were as usual for postage, in connection with the circulation of "The Mathematical Gazette" among members. This year there was also the postage on notices, sent to all secondary schools within reasonable distance, announcing the visit and public lecture of Dr. M. V. Wilkes from Cambridge in June, 1957, and the second and third general meetings referred to above.

The number of branch members is at present 45, which includes 2 Life Members of the Mathematical Association and 17 ordinary members of that Association. 6 new members have been admitted during the year, and there has been 1 transfer to Melbourne.

"The Mathematical Gazette" arrives regularly and is circulated as quickly as possible among the Associate Members. During the year, a copy of the Gazette for 1955 was placed in the Main University Library.

M. A. POPPLE, *Hon. Secretary*

17th March, 1958.

LEICESTER AND COUNTY BRANCH

REPORT FOR THE SESSION 1957-1958

Eight meetings were held during the session, all at the University. They were as follows:

"Enjoyment, Interest and Progress"—the Branch Presidential Address—by Mr. J. W. Hesselgreaves, Wyggeston Boys' School, Leicester.

The Annual Quiz between members of sixth forms of the Grammar Schools of the City and the County—a popular meeting which always attracts a large audience of sixth-formers and branch members.

"They Enjoy Maths." by Mr. Harold Fletcher of the Secondary School for Boys, Trench, Shropshire.

"The Use of Mathematics in Actuarial Work—How to become an Actuary" by Mr. K. J. Britt of the Britannic Assurance Company Limited, Birmingham.

The Branch Annual General Meeting, followed by a report by Miss F. E. Billsdon on the Oxford Mathematical Conference held at Trinity College in April, 1957 and a talk by Mr. W. Flemming on "Mathematics in Training Colleges."

"The Scope of Electronic Computers as applied to Scientific Problems" by Dr. C. M. Wilson of Messrs. Ferranti Ltd.

"Chasing x " by Professor T. A. A. Broadbent of the Royal Naval College, Greenwich—a meeting arranged primarily for members of sixth forms of Grammar Schools.

"Pure Mathematics for the Future Undergraduate" by Dr. E. J. F. Primrose of Leicester University.

The Modern Schools Discussion Group and the Primary Schools Discussion Group referred to in previous branch reports have continued to meet and this year, to complete the picture, we have formed a Grammar Schools Discussion Group which has so far held four meetings under the chairmanship of the Branch President.

There are now eighty-seven members of the Branch. Thirty-two of these are members of the Association.

Office holders for the session are as follows:

President, Mr. J. W. Hesselgreaves; *Vice-Presidents*, Professor R. L. Goodstein, Mr. W. Flemming, and Mr. T. R. Goddard; *Secretary*, Mr. W. E. Remington; *Treasurer*, Mr. L. G. Clarke.

W. E. REMINGTON, *Hon. Sec.*

REPORT OF THE COUNCIL FOR THE YEAR 1957

Membership

During the year ended 31st October, 1957, 353 ordinary members and 46 junior members were admitted to the Association. At the end of the year the membership figures were: Honorary, 5; Ordinary, 2,700; Junior, 130; Life, 249; a record total of 3,084 compared with 2,778 at the beginning of the year.

It is with regret that the Council reports the death of the following members: Mr. M. Adlard (1903), Mr. W. G. Borchardt (1902), Mr. J. S. Edwards (1910), Dr. T. G. Foulkes (1925), Mr. T. J. Garstang (1910), Mr. J. H. Hanson (1948), Miss L. E. Hardcastle (1925), Dr. G. B. Jeffery (1923), Dr. J. Miller (1912), Mr. H. V. Plum (1909), Dr. G. H. Wilson (1956).

Finance

The Treasurer's statement for the year ended 31st October, 1957, shows an excess of income over expenditure of £262 13s. 4d. Included in income, under the heading of donations, is the sum of £416 17s. This sum was subscribed by members, mainly life members,

in response to an appeal by the President. Without these donations the accounts would have shown an excess of expenditure over income of £154 3s. 8d., and a bank overdraft of about £180. This overdraft would have been increased during the two months which remained before the 1958 subscriptions began to come in.

The cash position was greatly fortified by loans amounting to £1,439 17s. 9d., also made by members in response to the President's appeal. These loans are due to be repaid in January, 1959.

The President's appeal, to which there was such a prompt and generous response, was launched primarily because the temporary suspension of the 7-year covenant scheme had made it impossible to finance the publication, in 1957-58, of the Higher Algebra Report and the Modern Schools Report. Even if it had been thought desirable to break into the £1,100 War Loan stock, the last cash reserve of the Association, the present is certainly not the time to consider selling this stock.

By the end of 1958 the long list of major publications undertaken at the end of the war will be completed, a decision on the 7-year covenant scheme will almost certainly have been announced by the Inland Revenue authorities, and the financial position should be sufficiently stable for long-term plans to be considered. The President's appeal was successful in making it possible to proceed without delay in the programme of publications.

The whole picture was subsequently changed by the handsome gift which the Association received from the Gulbenkian trustees, and the effect of this gift will be seen in the statement for 1957-58.

The Association has lost a Trustee through the death of Dr. G. B. Jeffery, and there are now only two Trustees, Mr. J. T. Cambridge and Mr. K. S. Snell.

The Mathematical Gazette

Distribution of the Gazette has again increased, both in this country and overseas. The need for more articles of immediate interest to teachers in schools has engaged the attention of the Editorial Board and it is hoped that measures taken in 1957 will bear fruit in 1958. Rising production costs can only be met by an increase in membership and the Gazette must play its part by aiming to interest a wider public.

Library

Recent accessions were listed in the December Gazette. This list does not include periodicals received in exchange for the Gazette which now form a substantial part of the Library. The 1957 Volume of Russian Mathematical Reviews has unfortunately not arrived and this has spoilt what promised to be an important new collection.

There has been a most disappointingly marked decline in borrowing from the Library, due in part perhaps to increased postal charges.

The Librarian would be glad to hear from members who have odd copies of the *Mathematical Gazette* Vols. I-IV for disposal.

The Teaching Committee

Activity has centred mainly on the work of two sub-committees. The Sixth Form Algebra Sub-Committee have seen their Report safely through the printer's hands and it has now been distributed to members. The Modern Schools Sub-Committee have completed their Report, which has been sent to the printer and may be expected during 1958. An additional meeting of the Teaching Committee was called in November to expedite the final stages of work on the Modern Schools Report; as this Report may be the biggest so far published by the Association, special consideration has been given to its form and distribution and recommendations will be made to Council on these points. Other sub-committees have continued their work.

A small sub-committee has been set up to consider the advisability of issuing a Report on the Teaching of Statistics.

The Primary Schools Report and the Trigonometry Report have been reprinted.

The Branches

The committee has met three times during the 1957-8 session and continues to be a useful forum for the interchange of branches' news. A new venture for the committee has been an effort to establish a closer link with branches overseas, one object being to provide, where desired, introductions and contacts for mathematical visitors to this country.

It is pleasant to be able to report that home branches continue their activities with vitality and variety; accounts of their meetings and proceedings are published regularly in the *Gazette*.

Problem Bureau

Requests for solutions continue to come in at the usual rate and are either answered by return of post or after a considerable delay. These requests sometimes come from sources outside the Association. The policy of the Bureau in such cases is to provide a solution if possible, unless it is a matter which should be referred to a professional consultant. While the Bureau can accept no payment, it is pointed out that the Association itself is open to receive donations.

There are still a few unsound questions set in public examinations, and they naturally tend to find their way to the Problem Bureau. While it is not the business of the Bureau to make representations

to the examining bodies concerned, due note is made of the various circumstances that arise.

The work of the Bureau is one of the many ways in which members of the Association can help one another, and it could be made more useful without placing further burdens on the present "staff." At present, solutions are filed only for the Cambridge College Entrance Scholarship Examinations in Mathematics. There are now many other scholarship and "A" level examinations of comparable importance, and it would be reasonable to file solutions to them as well. It is suggested that any member who is willing to help should undertake to send copies of the papers, together with solutions, of the Pure and Applied Mathematics papers of any one public examination in which he, or she, is interested.

To avoid overlapping, it is suggested that an initial enquiry be made to determine whether the examination is already covered. All communications for the Bureau should be addressed to Dr. G. A. Garreau, 90, Wyatt Park Road, London, S.W.2.

Officers and Council

Council wishes to record its thanks to the President, Mr. W. J. Langford and to the Officers for their work on behalf of members. The task of conducting the affairs of the Association is no light one; this year, in particular, has seen greatly enhanced activities, and the efforts of the President have been especially appreciated.

THE MATHEMATICAL ASSOCIATION

The fundamental aim of the Mathematical Association is to promote good methods of Mathematical teaching. Intending members of the Association are requested to communicate with one of the Secretaries. The subscription to the Association is 21s. per annum and is due on January 1st. Each member receives a copy of the *Mathematical Gazette* and a copy of each new Report as it is issued.

Change of address should be notified to the Membership Secretary, Mr. M. A. PORTER. If copies of the *Gazette* fail to reach a member for lack of such notification, duplicate copies can be supplied only at the published price. If change of address is the result of a change of appointment, the Membership Secretary will be glad to be informed.

Subscriptions should be paid to the Hon. Treasurer of the Mathematical Association.

The address of the Association and of the Hon. Treasurer and Secretaries is Gordon House, 29 Gordon Square, London, W.C.1.

A 5-YEAR SCHEME FOR THE ALTERNATIVE SYLLABUS

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FROM A REVIEW IN A.M.A. 'Why have the proposals of the Jeffery Report (for an Alternative Syllabus) made so little headway? It is due, I think, to the lack so far, of a really satisfying textbook based on its principles. . . . Mathematics Today will, in my view, prove a first-class advertisement for the Alternative Syllabus. The whole book is the result of an experiment in a girls' large grammar school, and I have no hesitation, although not myself a convert to the Jeffery Report, in describing this book as brilliant; the authors have certainly matched up to the new challenge and demonstrated in convincing style that a unified course is possible.

As a book written by a former senior mathematics mistress, and a head mistress of a girls' grammar school, it will nevertheless come as something of a shock that a text book on mathematics can be written, not ostensibly or so blatantly for boys, as such textbooks usually are, but to maintain the interest of both boys and girls.

I think no higher tribute can be given to the whole of this book than to say that future experiments and successes in the ideas of a unified course might well stand or fall on the way this textbook is received and adapted in the schools.'

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M.G. 83

AN INTRODUCTORY COURSE IN PURE MATHEMATICS

By K. B. SWAINE, M.A., Head of the Mathematics Department,
Yeovil School

This book is a continuation of the author's *Exercises in Elementary Mathematics*. It provides a Sixth Form course in Pure Mathematics to advanced and scholarship levels for science students, or to advanced level for mathematical specialists. The work, is, however, not fashioned to fit exactly the examination syllabus, and the opportunity has been taken, whenever it occurs, of pointing the way towards interesting developments of the topic in hand. With answers. 15s.

CALCULUS: Volumes I and II

By D. R. DICKINSON, M.A., Ph.D.(Cantab.), Head of the
Mathematical Department, Bristol Grammar School

The author has provided a course in Calculus which may be studied by the pupil with a minimum of help and guidance from the teacher. While concerned primarily with manipulative techniques the treatment is sufficiently rigorous to meet all present-day requirements. The first volume covers 'A' level G.C.E. and most scholarship syllabuses and the second volume continues the study of calculus up to and, in the later chapters, somewhat beyond the level required for all scholarship examinations. These later chapters have been added to increase the general utility of the book and to make it suitable for a first course at university level. With answers. Part I, 17s. 6d.
Part II, 13s. 6d.

A FIRST GEOMETRY: Part III

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